VECTOR METHOD: KINETIC PARAMETERS ANALYSIS AND IMPACTS TO BEARINGS OF THE COUPLED ROTOR DYNAMICS

dedicated in memory of Academician Konstantin Vasiljevich Frolov (1932-2007)

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Abstract. ***A*** *survey as review of author's published research results in area of vector method with applications in Classical Mechanics is presented. In the first part of the paper mass moment vectors, introduced and presented by author at ICTAM Haifa 92, are used to present a vector method suitable for application to the kinetic parameter analysis of coupled rigid rotor dynamics.*

By use vector $\vec{\mathsf{J}}_A^{(\vec{n})}$ *of he body mass inertia moment at the point for the axis oriented by the unit vector and corresponding vector* $\vec{\mathsf{D}}_A^{(\vec{n})}$ *of the rotation rigid body mass deviational moment as well as the pure kinematic vector rotators orthogonal to corresponding rotation axis, the kinetic pressures as well as kinetic impacts to the heavy multi step coupled rotor bearings are analytically expressed.*

It is clearly visible, that pair of two components of the kinetic pressures as well as kinetic impacts to the every of the one shaft bearings from the set of the coupled rotors are equal intensity and opposite directions and both of one pair build ***a deviational couple of the kinetic impacts*** *with intensity* $\left|\vec{\mathsf{D}}_{A(j)}^{(\vec{n})}\right|\left|\int_{t_0}^{t}\vec{\mathsf{R}}_j dt\right|$ *to the every pair of the shaft bearings of every of the coupled rotor shafts.*

Key words: *Vector method, mass inertia moment vectors, deviational mass moment vector, kinematic vector rotator, kinetic parameters, coupled rotors, coupled rotations, deviational couple, kinetic pressures on bearings, impact forces, kinetic impact to the bearing, trigger, coupled singularities, bifurcation,* ***h****omoclinic orbit, separatrice layering.*

I. Introduction

The interest in the study of vector and tensor methods with applications in the ***Dynamics*** especially in ***Kinetics*** of rigid and solid body coupled rotational motions and deformation displacements as a new qualitative approach to the optimization time study process grew exponentially over the last few years because of theory challenges involved in the study of technical sciences need such optimization of university systems study. Optimization of the time for introducing new basic high level scientific ideas (logic and philosophical) which are easy to understand to most of students in the study process and for engineering applications is very important.

A series of author's published research results in the area of vector method with applications in Classical Mechanics is presented in the monograph [2] and [3], as well as in a series of papers and scientific congresses published communications [1, 4-33]. In the foundation of this such vector method, suitable for applications to the rotors as well as to the coupled rotor kinetics, are definitions of mass moment vectors coupled for the pole and the axis. Main of these vectors is $\vec{\mathsf{J}}_O^{(\vec{n})}$ of the body mass iner-

tia moment at the point $A = O$ for the axis oriented by the unit vector $\vec{n}$: $\vec{J}_A^{(\vec{n})} \overset{def}{=} \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]dm$, and his corresponding component $\vec{D}_A^{(\vec{n})}$ vector of the rotation rigid body mass deviational moment of the rotation axis through the spherical bearing A and for the rotation axis and two vector rotators, pure kinematical vectors - rotators orthogonal to the rotation axis and in the deviational plane through the axis and with intensity $R = \sqrt{\dot{\omega}^2 + \omega^4}$.

In the listed paper many applications of the discovered vector method by using mass moment vectors [1-8], [13] and [21], are presented. The expressions of kinetic parameters of heavy rotors kinetics [6,8,10,11,17] as well as of coupled multi step rotors [9,15, 16, 18, 19, 20, 26-31] and gyro rotors [20, 24, 25, 29,30.] are obtained..

Analogy between models of stress state, strain state and state of the body mass inertia moments are considered and presented in a series of References [7, 11]. A trigger of coupled singularities, on an example of coupled rotors with deviational material particles are presented in References [18,19]. Nonlinear phenomena in rotor dynamics as it is were investigated in the series of References [26-30]. These nonlinear phenomena include: phase portraits and homoclinic orbits visualization of nonlinear dynamics of multiple step reductor/multiplier; nonlinear dynamics of planetary reductor with turbulent damping; nonlinear dynamics of a heavy material particle along circle which rotates and optimal control; and homoclinic orbits layering in the coupled rotor nonlinear dynamics and chaotic clock models. Kinematical vector rotators are presented in Refs. [1,31].

From time to time it is useful to again pay attention to classical models of dynamics of mechanical systems and evaluate possibilities for new approaches to these classical results by using other than methods usually used in the classical literature.

In present papers mass moment vectors and their deviational components are used to determine kinetic pressures as well as kinetic impacts on shaft bearings of coupled rotor dynamics in the earth's gravity field.

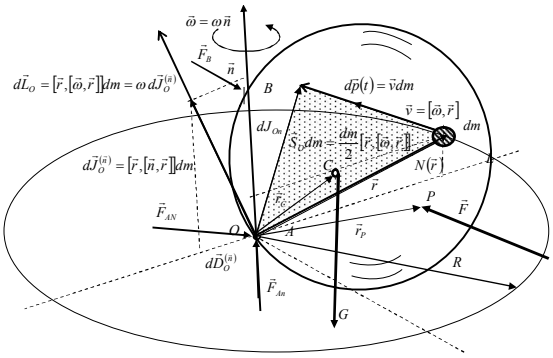
II. Mass moment vectors for the pole and axis – A new vector view to Classical Mechanics

The monograph [2] and monograph paper [3] contain definitions of three kinetic vectors fixed to a certain point and axis passing through the given space body point as a referent pole.

Definitions of these mass moments vectors for the pole and the axis: Vector $\vec{M}_O^{(\vec{n})}$ of the body mass, Vector $\vec{S}_O^{(\vec{n})}$ of the body mass static (linear) moment, Vector $\vec{J}_O^{(\vec{n})}$ of he body mass inertia moment for the axis, oriented by the unit vector $\vec{n}$, thought the point – pole O, are in the following form:

$$\vec{M}_O^{(\vec{n})} \overset{def}{=} \iiint_V \vec{n}\,dm = M\vec{n}, \qquad \vec{S}_O^{(\vec{n})} \overset{def}{=} \iiint_V [\vec{n},\vec{\rho}]dm, \qquad \vec{J}_O^{(\vec{n})} \overset{def}{=} \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]dm \qquad dm = \sigma dV; \quad (1)$$

where $\vec{\rho}$ is the position vector of the elementary body mass dm with respect to the common pole O (see Figure 1. a*). For special cases, details can be seen in Refs. [1-8].



a* **b*** **c***

Figure 1. **a*** Graphical presentation of the kinetic reactions to the rotor (rigid body) shaft bearings on the elementary mass particle dm of the rotate rigid body
b* A discus respect to the axis of the shaft in an eccentrically skew position, and the graphical presentation of the vector of the disc mass inertia moment for the reference point and an oriented axis of the shaft and of the corresponding deviational plane.
c* A graphical presentation of the Theorem of material body mass inertia moment vectors for the two parallel axes through two referential points..

In the case when the pole O_1 is the centre C of the body mass the vector $\vec{r}_C$ (the position vector of the masses center with respect to the pole O_1) is equal to zero, whereas the vector $\vec{\rho}_O$ turns into $\vec{\rho}_C$ so the relation between tro mass inertia moment vectors for the parallel axis through different poles can be written in the following form (see Figure 1.c*):

$$\vec{\mathrm{J}}_O^{(\vec{n})} = \vec{\mathrm{J}}_C^{(\vec{n})} + [\vec{\rho}_C,[\vec{n},\vec{\rho}_C]]\mathrm{M} \qquad (2)$$

This expression (2) *represents the mathematical and vector forms of the theorem of change(rate) of the mass inertia moment vector for the pole and the axis, when the axis is translated from the pole in the mass center* C *to the arbitrary point, pole* O.

The *Huygens-Steiner theorems* (see Ref. [2]) for the body mass axial inertia moment as well as for the mass deviational moments *came from this theorem* (2) about the change of the vector $\vec{\mathrm{J}}_O^{(\vec{n})}$ of the body mass inertia moment at the point O for the axis oriented by the unit vector $\vec{n}$ passing trough the mass center C and when the axis is moved translator to the other point O.

III. Linear and angular momentum of the rigid body rotation

Figure 1 a* shows the rigid body with the fixed rotation axis, through fixed point A, around which it rotates with the angular velocity $\vec{\omega}$ which changes in time so that angular acceleration $\dot{\vec{\omega}}$ appears. Kinetic energy is expressed as $2E_k = \omega\left(\vec{\omega}, \vec{J}_A^{(\vec{n})}\right) = \omega^2 J_A^{(\vec{n})}$. For this case of the rigid body rotation around fixed axis, the linear momentum and angular momentum are in the following forms:

$$\vec{K} = [\vec{\omega}, \vec{\rho}_C]M = \omega\vec{S}_A^{(\vec{n})}, \qquad \vec{L}_A = \vec{\omega}\left(\vec{n}, \vec{J}_A^{(\vec{n})}\right) + \omega\left[\vec{n}\left[\vec{J}_A^{(\vec{n})}, \vec{n}\right]\right] = \vec{\omega}\left(\vec{n}, \vec{J}_A^{(\vec{n})}\right) + \omega\vec{D}_A^{(\vec{n})}. \quad (3)$$

Since the velocity $\vec{v}$ and the acceleration $\vec{a}$ of the body elementary mass at the point N are: $\vec{v} = [\vec{\omega}, \vec{\rho}]$, $\vec{a} = \left[\dot{\vec{\omega}}, \vec{\rho}\right] + [\vec{\omega}, [\vec{\omega}, \vec{\rho}]]$, then the main vector $\vec{F}_{rj}$ of the inertia force and vector $\vec{M}_{Aj}$ of the main reduction moment of the inertia forces of the overall rigid body rotating around the axis with the angular velocity $\vec{\omega}$ are in the following form:

$$\vec{F}_{rj} = -\iiint_V \vec{a}\,dm = -\dot{\omega}\vec{S}_A^{(\vec{n})} - \omega\left[\vec{\omega}, \vec{S}_A^{(\vec{n})}\right], \qquad \vec{M}_{Aj} = \iiint_V \left[\vec{\rho}, d\vec{F}_{rj}\right] = -\dot{\omega}\vec{J}_A^{(\vec{n})} - \omega\left[\vec{\omega}, \vec{J}_A^{(\vec{n})}\right] \quad (4)$$

For the case of the rigid body rotation around fixed axis, under actions of active forces $\vec{F}_k$, $k = 1,2,3,...,N$, with vector positions $\vec{\rho}_k$, $k = 1,2,3,...,N$ of the points of application on the rigid body, we can write the following - two vector equations of the rigid body dynamic equilibrium:

$$\frac{d\vec{K}}{dt} = \left|\vec{S}_A^{(\vec{n})}\right|\left(\dot{\omega}\vec{u}_1 + \omega^2\vec{v}_1\right) = \vec{R}\left|\vec{S}_A^{(\vec{n})}\right| = R\left|\vec{S}_A^{(\vec{n})}\right|\vec{r}_1 = \sum_{k=1}^{k=N}\vec{F}_k + \vec{F}_A + \vec{F}_B, \quad (5)$$

$$\frac{d\vec{L}_A}{dt} = \dot{\vec{\omega}}J_A^{(\vec{n})} + \dot{\omega}\vec{D}_A^{(\vec{n})} + \omega\left[\vec{\omega}, \vec{D}_A^{(\vec{n})}\right] = \dot{\vec{\omega}}J_A^{(\vec{n})} + \left|\vec{D}_A^{(\vec{n})}\right|\vec{R} = \sum_{k=1}^{k=N}\left[\vec{\rho}_k, \vec{F}_k\right] + \left[\vec{\rho}_B, \vec{F}_B\right]. \quad (6)$$

where $\vec{R}_1 = R\vec{r}_1$, $R = \sqrt{\dot{\omega}^2 + \omega^4}$, $\vec{F}_{AN}$ and $\vec{F}_B$ are kinetic pressures on the rotating rigid body bearings and $\vec{r}_B$ vector position bearing B with relation to the hinged bearing A.

The vector rotator $\vec{R} = R\vec{r}$, for the rigid body rotation axis and pole A, which is rotating and increasing by the angular velocity $\vec{\omega}$ and by the angular acceleration $\dot{\vec{\omega}}$, and, at the same, causes the inertia forces deviational moment to increase. The vector rotator $\vec{R} = R\vec{r}$ and the vector rotator $\vec{R}_1 = R\vec{r}_1$ are orthogonal to one another, and rotate with the same angular velocity and angular acceleration:

$$\dot{\gamma} = \omega + \dot{\vartheta} = \omega\frac{\omega^4 + \ddot{\omega}\omega - \dot{\omega}^2}{\omega^4 + \dot{\omega}^2}, \quad \ddot{\vartheta} = \ddot{\omega}\omega^2\frac{\left(\omega^4 + \dot{\omega}^2\right)}{\left(\omega^4 + \dot{\omega}^2\right)^2} + \frac{\left(+6\dot{\omega}^3\omega^4 - 6\omega^5\dot{\omega}\ddot{\omega} - 2\dot{\omega}^5 + 2\omega\dot{\omega}^3\ddot{\omega} - 2\dot{\omega}\ddot{\omega}^2\omega^2\right)}{\left(\omega^4 + \dot{\omega}^2\right)^2} \quad (7)$$

Also, we can write the following relations:

$$\frac{\left|\vec{F}_{rj}\right|}{\left|\vec{M}_A^{(dev)}\right|} = \frac{\left|\frac{d\vec{K}}{dt}\right|}{\left|\frac{d\vec{L}_A^d}{dt}\right|} = \frac{\left|\vec{S}_A^{(\vec{n})}\right|}{\left|\vec{D}_A^{(\vec{n})}\right|} = const. \quad (8)$$

IV. Kinetic pressures and kinetics impacts on shaft bearings excited by the body rotation

For kinetic reactions (pressures), $\vec{F}_{AN}$ and $\vec{F}_B$, in the rotor bearings it is possible to write the following simple expressions (for detail see Ref. [2]):

$$\vec{F}_{AN(kin)} = \left|\vec{S}_O^{(\vec{n})}\right|\vec{\mathsf{R}}_1 - \frac{1}{r_B}\left|\vec{\mathsf{D}}_O^{(\vec{n})}\right|\vec{\mathsf{R}} \quad , \qquad \vec{F}_{B(kon)} = \frac{1}{r_B}\left|\vec{\mathsf{D}}_O^{(\vec{n})}\right|\vec{\mathsf{R}} \tag{9}$$

From previous expression for kinetic reactions (pressures) in the rotor bearings it is clearly visible that there are two parallel and opposite direction components $\pm\frac{1}{r_B}\left|\vec{\mathsf{D}}_O^{(\vec{n})}\right|\vec{\mathsf{R}}$, on the distance r_B, which constitute **a deviational couple of kinetic pressures** with intensity $\left|\vec{\mathsf{D}}_A^{(\vec{n})}\right|\left|\vec{\mathsf{R}}\right|$.

From expressions for the impulses of the impact forces to the rotating body bearings $\vec{K}_{F_{AN}}$ and $\vec{K}_{F_B}$, we can separate two parts depending on kinetic parameters of rigid body dynamics in following forms (see Refs. [32,33]):

$$\vec{K}_{F_{AN}DEV} = -\frac{1}{r_B}\left|\vec{\mathsf{D}}_O^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathsf{R}}\,dt \quad , \qquad \vec{K}_{F_BDEV} = \frac{1}{r_B}\left|\vec{\mathsf{D}}_O^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathsf{R}}\,dt = -\vec{K}_{F_{AN}DEV} \tag{11}$$

which are parallel in the direction $\pm\int_{t_0}^{t_0+\tau}\vec{\mathsf{R}}\,dt = \pm\int_{t_0}^{t_0+\tau}\left[\dot{\omega}\vec{u} + \omega^2\vec{w}\right]dt = \pm\left(\omega_0 + \frac{\left(\vec{n},\left[\vec{r}_P,\vec{K}_{F_{wd}}\right]\right)}{\left(\vec{\mathsf{J}}_O^{(\vec{n})},\vec{n}\right)}\right)\vec{u}$ and equal intensity and opposite direction. These two deviational components of impulses of impact forces to the rotating body bearings build a deviational impact couple with intensity $\mathsf{M}^{dev}_{\vec{K}(\vec{F}_A,\vec{F}_B)} = \left|\vec{\mathsf{D}}_O^{(\vec{n})}\right|\left|\int_{t_0}^{t}\vec{\mathsf{R}}\,dt\right|$.

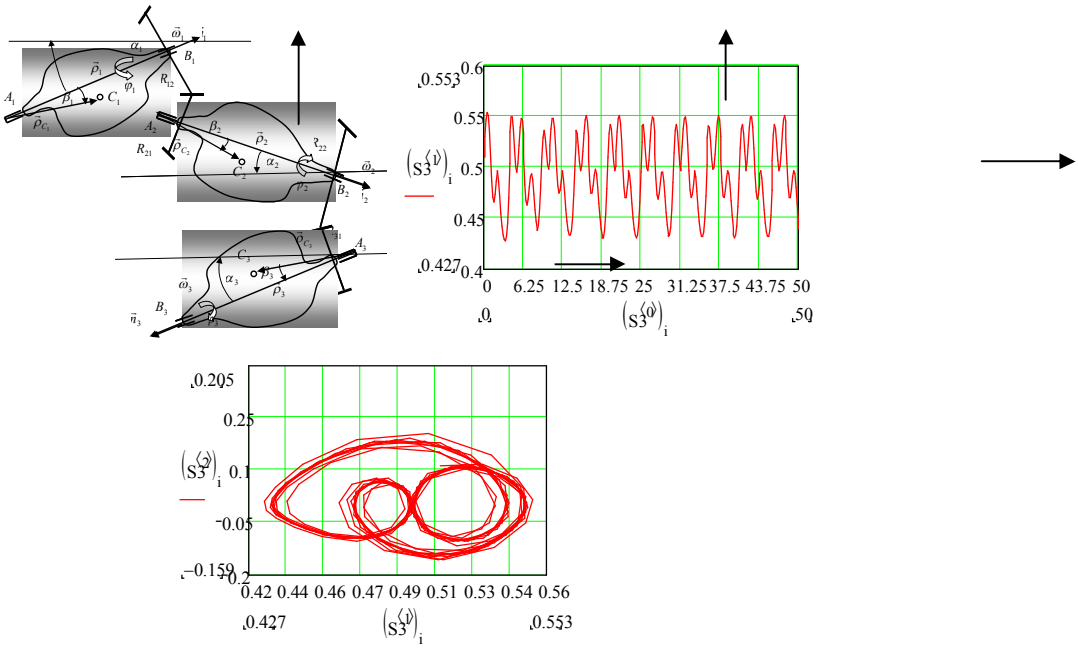
This couple depends only on the rigid body mass distribution around axis of rotation and angular velocity and angular acceleration during time interval $\Delta t = t - t_0 = \tau \to 0$ in accordance with corresponding difference equation of the impact rotation.

This is impact deviational couple to the rotating body bearings and in engineering practice with very dangerous effect, which can destroy shaft bearings.

V. Basic vector equations of coupled heavy rotor dynamics and kinetic pressures/impacts to the bearings

In Figure 2.a*, a system of coupled heavy rotors with deviation properties of rotate mass distribution is presented. In this Figure we can see the basic kinematic and kinetic parameters data with geometric data, as well as angular coordinates (for detail see Ref. [24]). We introduce the following denotations: $\vec{\mathsf{J}}_{\vec{n}_i}^{(A_i)}, i = 1,2,3,...,M$ are mass inertia moment vectors of heavy coupled rotors for the corresponding axis oriented by unit vectors $\vec{n}_i$ and for corresponding pole in the fixed-hinged bearing denoted with A_i ; C_i, $i = 1,2,3,...,M$ - are mass centers of the heavy rotors; $\vec{\rho}_{C_I}$ - are mass center C_i position vectors with respect to the fixed-hinged bearing denoted with A_i ; β_i -are angles, formed between the mass center C_i position vectors $\vec{\rho}_{C_I}$ and the corresponding rotor shaft axis; α_i - are angles

between axis of the i-th rotor shaft and horizontal plane (or corresponding horizontal axis); B_i, $i=1,2,3,...,M$ - are movable (cylindrical) bearings on the corresponding rotor shaft; $R_{i1}, R_{i2}, i=1,2,3,...,M$ - are radii of the basic circles of the corresponding gears on the i-th rotor shaft, near to the fixed-hinged bearing A_i, or cylindrical bearing B_i; $\vec{\rho}_i$, $i=1,2,3,...,M$ are position vectors of cylindrical bearings B_i with respect to the corresponding fixed-hinged bearing A_i on the corresponding rotor shaft.



a* **b*** **c***

Figure 2. Three-step coupled heavy rotors (a*) forced nonlinear dynamics visualizations. (reductor and multipliers) . b* Time-history curves; and c* Phase trajectory portraits of the forced nonlinear dynamics.

Let us denote with $\varphi_i, i=1,2,3,...,M$ the angular coordinates of the corresponding rotor rotation, ad with $\omega_i=\dot{\varphi}_i$, $i=1,2,3,...,M$ the corresponding angular velocities. Then the corresponding ratio:

$$i_{i,(i+1)}=\frac{\omega_{i+1}}{\omega_i}=\frac{\dot{\varphi}_{i+1}}{\dot{\varphi}_i}=\frac{R_{i2}}{R_{(i+1),1}}=\frac{1}{k_{i,(i+1)}} \qquad (12)$$

are reduction or multiplier factors between i-th and $(i+1)$-th coupled rotors by light, neglected mass gears.

Firstly, it is necessary to conduct the decomposition of the rotor system and of the equivalent forces of mutual influence between rotor dynamics introduced by contact forces between gears of the previous and the next rotors, which we denote with:

$\vec{\mathbf{F}}_{i1}=\mathbf{F}_{i1}\vec{u}_{i1}, i=1,2,3,...,(M-1)$ and $\vec{\mathbf{F}}_{i2}=\mathbf{F}_{i2}\vec{u}_{i2}, i=1,2,3,...,(M-1),M$,

where $\mathbf{F}_{11}=0, \mathbf{F}_{M2}=0$, $\mathbf{F}_{i1}=\mathbf{F}_{(i-1)2}$, $\vec{u}_{i1}=-\vec{u}_{(i-1)2}$, and i is the order of the shaft in the series of coupled rotors, and M is the number of coupled rotor shafts in the system of coupled heavy rotors dynamics.

Now, for each rotation of the heavy rotors we write vectors of the linear momentum $\vec{\mathsf{K}}_{\vec{n}_i}^{(A_i)}$ and angular momentum $\vec{\mathsf{L}}_{\vec{n}_i}^{(A_i)}$ for the pole in the corresponding fixed-hinged bearing A_i on the corresponding rotor shaft axis oriented by unite vector $\vec{n}_i$ in acordance with expressions (3) and (4) from part III (and also see Reference [24]):

$$\vec{\mathsf{K}}_{\vec{n}_i}^{(A_i)} = \omega_i \vec{\mathsf{S}}_{\vec{n}_i}^{(A_i)}, \quad \vec{\mathsf{L}}_{\vec{n}_i}^{(A_i)} = \omega_i \vec{\mathsf{J}}_{\vec{n}_i}^{(A_i)} = \vec{\omega}_i \left(\vec{\mathsf{J}}_{\vec{n}_i}^{(A_i)}, \vec{n}_i \right) + \omega_i \vec{\mathsf{D}}_{\vec{n}_i}^{(A_i)} \qquad , \; i = 1,2,3,...,M \tag{13}$$

where:

$$\vec{\mathsf{S}}_{\vec{n}_i}^{(A_i)} \overset{def}{=} \iiint_V [\vec{n}_i, \vec{\rho}] dm \quad , \; \vec{\mathsf{J}}_{\vec{n}_i}^{(A_i)} \overset{def}{=} \iiint_V [\vec{\rho}, [\vec{n}_i, \vec{\rho}]] dm \, , \qquad \vec{\mathsf{D}}_{\vec{n}_i}^{(A_i)} \overset{def}{=} [\vec{n}_i, [\vec{\mathsf{J}}_{\vec{n}_i}^{(A_i)}, \vec{n}_i]] \quad , \; i = 1,2,3,...,M \tag{14}$$

Differential equations of the free rotations of decomposed rotors are given in the following forms (see Ref. [24]) :

$$\left(\vec{\mathsf{J}}_{\vec{n}_i}^{(A_i)}, \overset{*}{\vec{\omega}}_i \right) - \left(\vec{n}_i, \left[\vec{\rho}_{C_i}, \vec{\mathbf{G}}_i \right] \right) = \left(R_{i1} \mathbf{F}_{i1} - R_{i2} \mathbf{F}_{i2} \right) \qquad i = 1,2,3,...,M \tag{15}$$

Kinetic pressures on the corresponding rotor shaft bearings are:

$$\begin{aligned} \vec{\mathbf{F}}_{A_i}^{(\vec{n}_i)} &= \vec{n}_i \left(\vec{n}_i, \vec{\mathbf{F}}_{A_i} \right) = -\vec{n}_i \left(\vec{n}_i, \vec{\mathbf{G}}_i \right) \\ \vec{\mathbf{F}}_{Ai}^{(N)} &= \vec{\mathsf{R}}_i \left| \vec{\mathsf{S}}_{\vec{n}_i}^{(A_i)} \right| - \vec{\mathbf{F}}_{Bi} - \vec{\mathbf{F}}_{i1} - \vec{\mathbf{F}}_{i2} - \sum_{k=1}^{k=K_i} \left[\vec{n}_i, \left[\vec{\mathbf{F}}_{ki}, \vec{n}_i \right] \right] - \left[\vec{n}_i, \left[\vec{\mathbf{G}}_i, \vec{n}_i \right] \right] \\ \vec{\mathbf{F}}_{Bi} &= \frac{1}{\rho_{B_i}} \vec{\mathsf{R}}_i \left| \vec{\mathsf{D}}_{\vec{n}_i}^{(A_i)} \right| + \mathbf{F}_{i2} \vec{u}_{i2} + \frac{1}{\rho_{B_i}} \left[\vec{n}_i, \left[\left[\vec{\rho}_{C_i}, \vec{\mathbf{G}}_i \right], \vec{n}_i \right] \right] \qquad i = 1,2,3,...,M \end{aligned} \tag{16}$$

in which we introduce following notations for kinematic vector rotators:

$$\mathsf{R}_i = \sqrt{\dot{\omega}_i^2 + \omega_i^4}, \quad \vec{\mathsf{R}}_i = \mathsf{R}_i \vec{r}_i \qquad i = 1,2,3,...,M \tag{17}$$

From previous expression for kinetic reactions (pressures) in the rotor system bearings it is clearly visible that there are two parallel and opposite direction components $\pm \frac{1}{r_B} \left| \vec{\mathsf{D}}_{\vec{n}_i}^{(A_i)} \right| \vec{\mathsf{R}}_i$, on the distance r_B, which constitute **a deviational couple of kinetic pressures** with intensity $\left| \vec{\mathsf{D}}_{\vec{n}_i}^{(A_i)} \right| \left| \vec{\mathsf{R}}_i \right|$.

By eliminating contact forces from the system of differential equations (15) by multiplying with R_{ik} or with $1/R_{ik}$ and summarizing with the sum of previous equations and by introducing coefficients of reductions or multiplications $k_{i(i+1)} = \frac{R_{(i+1)1}}{R_{i2}} = i_{i(i+1)}$, we obtain one differential equation with respect to the one angle of rotation/oscillation, for example angle of rotation of the first rotor, or M -the or other arbitrary rotor from the system of the coupled rotors. For the system of M coupled heavy rotors (for $(M-1)$ step coupled system):

$$\ddot{\varphi}_1 + \Omega_{1M}^2 \left\{ \sin \varphi_1 + \left[\sum_{i=2}^{i=M} \frac{\prod_{j=1}^{j=i} i_{j,(j+1)}}{\prod_{j=1}^{j=i} \lambda_{j,(j+1)}} \nu_{1i}^2 \sin \left(\prod_{j=1}^{j=i} i_{j,(j+1)} \varphi_1 \right) \right] \right\} = 0 \tag{18}$$

where:

$$\Omega_{1M}^2 = \frac{\lambda_{1M}\Omega_1^2}{\lambda_{(M-1)M}\Delta_{1,(M-1)} + \prod_{j=1}^{j=M} i_{j,(j+1)}^2};\ \Delta_{11} = \lambda_{12} + i_{12}^2 \qquad (19)$$

The relations between generalized angle velocity and velocity of the k-th rotor shaft is:

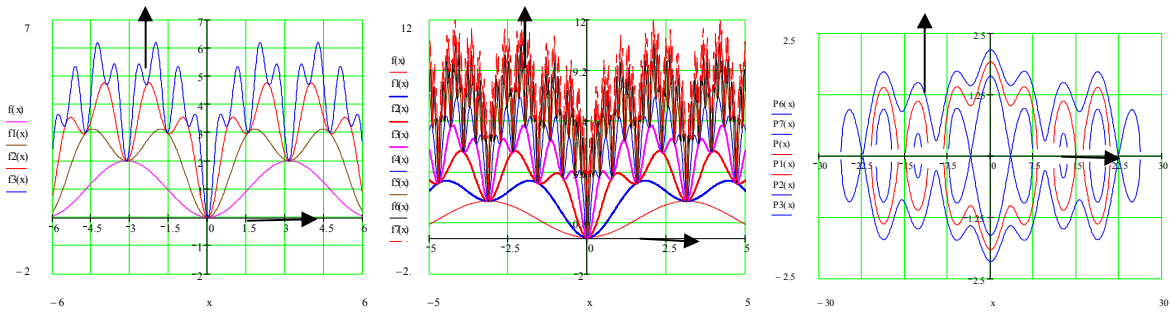
$$\omega_k = \dot{\varphi}_k = i_{1k}\dot{\varphi}_1 = \prod_{j=1}^{j=k} i_{j(j+1)}\dot{\varphi}_1 = \frac{1}{k_{1k}}\dot{\varphi}_1 = \prod_{j=1}^{j=k}\frac{1}{k_{j(j+1)}}\dot{\varphi}_1 = \prod_{j=1}^{j=k}\frac{1}{k_{j(j+1)}}\omega_1 \qquad (20)$$

Differential equations of the coupled rotor dynamics in the field of gravity is in the following form:

$$\sum_{i=1}^{i=M}\left\{\left(\prod_{j=1}^{j=n-i} k_{j,(j+1)}\right)\left[\left(\vec{\mathrm{J}}_{\vec{n}_i}^{(A_i)}, \vec{\omega}_i^*\right) - \left(\vec{n}_i, \left[\vec{\rho}_{C_i}, \vec{\mathbf{G}}_i\right]\right)\right]\right\} = 0 \qquad (21)$$

If we introduce the following denotation:

$$\Omega_i^2 = \frac{M_i g \rho_{C_i}\sin\beta_i\cos\alpha_i}{\mathbf{J}_{\vec{n}_i}^{(A_i)}} = \frac{g}{\ell_{red,i}},\quad \ell_{red,i} = \left[\frac{\left(\mathbf{i}_{\vec{n}_i}^{(C_i)}\right)^2}{\rho_{C_i}\sin\beta_i} + \rho_{C_i}\sin\beta\right]\frac{1}{\cos\alpha_i} \qquad (22)$$



a* **b*** **c***

Figure 3. Multistep-coupled heavy rotors free nonlinear dynamics vizualizations (Multipliers).a* and b* Potential energy curves families. c* Phase homoclinic trajectories potraits (constante energy curves). Homoclinic orbits layering for conservative system.

From expressions for the impulses of the impact forces to the rotating body bearings $\vec{K}_{F_{AN}}$ and $\vec{K}_{F_B}$ presented in the previous part by (11) and also by transforming expressions (16) using analogy, we can write the kinetic impacts of the impulses of the impact forces to the every rotor shaft bearings $\vec{K}_{F_{AN},i}$ and $\vec{K}_{F_B,i}$ from the set of the coupled rotor system in the following forms:

$$\vec{K}_{A_i}^{(\vec{n}_i)} = \vec{n}_i\left(\vec{n}_i, \vec{K}_{A_i}\right) = -\vec{n}_i\sum_{k=1}^{k=K_i}\left(\vec{n}_i, \vec{K}_{ki}\right)$$

$$\vec{K}_{Ai}^{(N)} = \left|\vec{\mathrm{S}}_{\vec{n}_i}^{(A_i)}\right|\int_{t_0}^{t}\vec{\mathrm{R}}_{1,i}dt - \vec{K}_{Bi} - \vec{K}_{i1} - \vec{K}_{i2} - \sum_{k=1}^{k=K_i}\left[\vec{n}_i, \left[\vec{K}_{ki}, \vec{n}_i\right]\right] \qquad (23)$$

$$\vec{K}_{Bi}=\frac{1}{\rho_{B_i}}\left|\vec{\mathrm{D}}_{\vec{n}_i}^{(A_i)}\right|\int_{t_0}^{t}\vec{\mathrm{R}}_i dt+\frac{1}{\rho_{B_i}}\sum_{k=1}^{k=K_i}\left[\vec{n}_i,\left[\left[\vec{\rho}_{k,i},\vec{K}_{k,i}\right],\vec{n}_i\right]\right]+\vec{K}_{i2}\vec{u}_{i2}$$

where

$$\sum_{i=1}^{i=M}\left\{\left(\prod_{j=1}^{j=n-i}k_{j,(j+1)}\right)\left[\left(\vec{\mathrm{J}}_{\vec{n}_i}^{(A_i)},\vec{\omega}_i^{*}\right)-\left(\vec{n}_i,\left[\vec{\rho}_{C_i},\vec{K}_i\right]\right)\right]\right\}=0,\quad \omega_k=\dot{\varphi}_k=\prod_{j=1}^{j=k}\frac{1}{k_{j(j+1)}}\omega_1 \tag{24}$$

VI. Equations of phase trajectories of the coupled heavy rotors nonlinear dynamics

By integrating the differential equations (18) for the homogeneous case, the equations of the phase trajectories take the following forms:

$$\dot{\varphi}_1^2-\dot{\varphi}_{01}^2-2\Omega_{1M}^2\sum_{k=0}^{k=M}\left[\cos i^k\varphi_1-\cos i^k\varphi_{01}\right]=0 \tag{25}$$

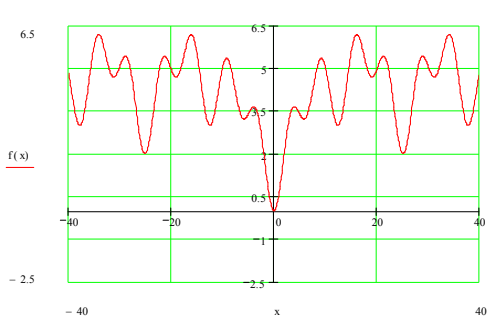
VII. Numerical experiment and graphical presentations of the nonlinear phenomena of coupled rotors dynamics

Numerical experiment on the expressions of phase trajectories and the constant energy curves as well as on the expression of the vector rotator are pointed out for different steps of the system and for different coefficients of reduction and multiplications. For numerical experiment the following expressions for velocity

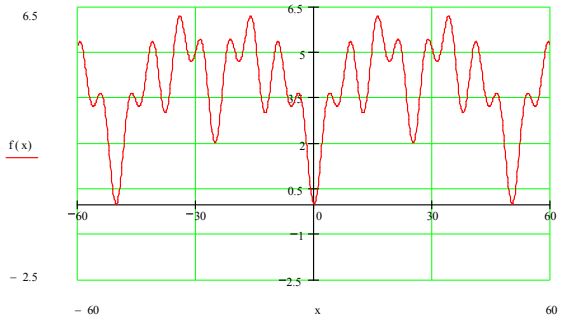
$$v_1=\pm\sqrt{v_{01}^2+\sum_{k=0}^{k=M}\left[\cos i^k\varphi_1-\cos i^k\varphi_{01}\right]}\quad,\quad v_1=\frac{\dot{\varphi}_1}{\sqrt{2}\Omega_{1M}} \tag{26}$$

and for vector rotaor intensity

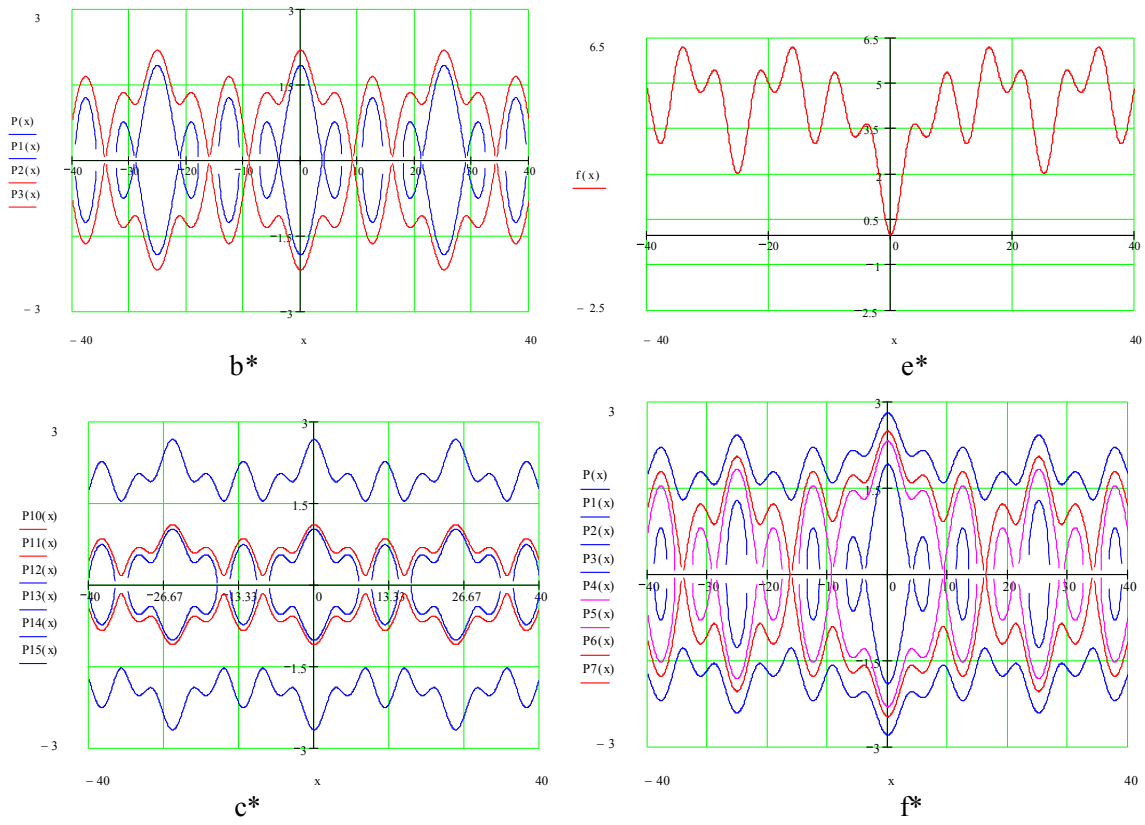
$$\tilde{\mathrm{R}}_j=\sqrt{\frac{1}{2}\left[\sum_{k=0}^{k=M}i^k\sin\left(i^k\varphi_1\right)\right]^2+i^j\left\{v_{01}^2+\sum_{k=0}^{k=M}\left[\cos i^k\varphi_1-\cos i^k\varphi_{01}\right]\right\}^2},\quad \tilde{\mathrm{R}}_j=\frac{\mathrm{R}_j}{2\Omega_{1M}^2 i^j} \tag{27}$$



a*



d*

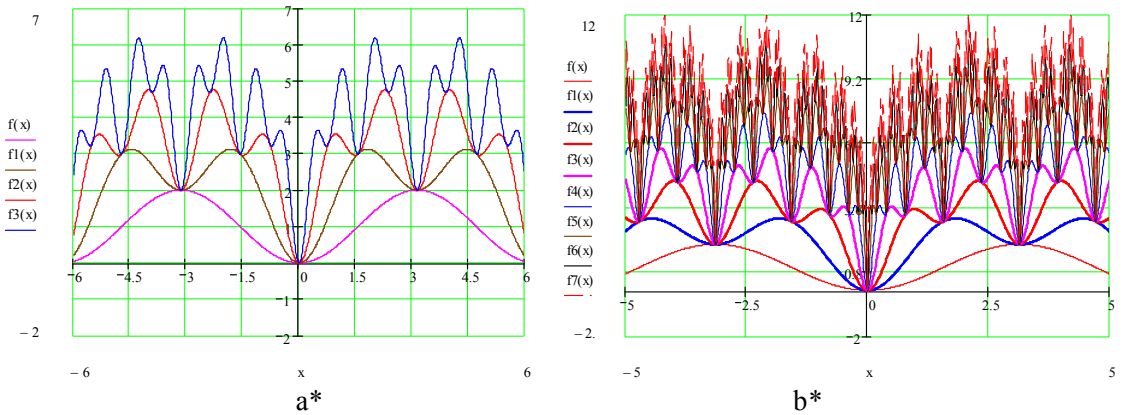


b* e* c* f*

Figure 4. Four-step coupled heavy rotors nonlinear dynamics visualizations (reductor k=1/2). a*, d* and e* Potential energy curves. b* and f* Phase trajectories portraits. Homoclinic orbits. c* Rotator family curves.

$k := 1 \quad N := 1,2,3,4,5 \quad k1 := 2 \quad k2 := 2 \quad k3 := 2 \quad k4 := 2 \quad k5 := 2$

$$f7(x) := \left[1 - \cos(x) + (1 - \cos(k5 \cdot x)) + \left(1 - \cos\left(k5^2 \cdot x\right)\right) + \left(1 - \cos\left(k5^3 \cdot x\right)\right) + \left(1 - \cos\left(k5^4 \cdot x\right)\right) + \left[\left(1 - \cos\left(k5^5 \cdot x\right)\right) + \left(1 - \cos\left(k5^6 \cdot x\right)\right) + \left(1 - \cos\left(k5^7 \cdot x\right)\right)\right]\right]$$



a* b*

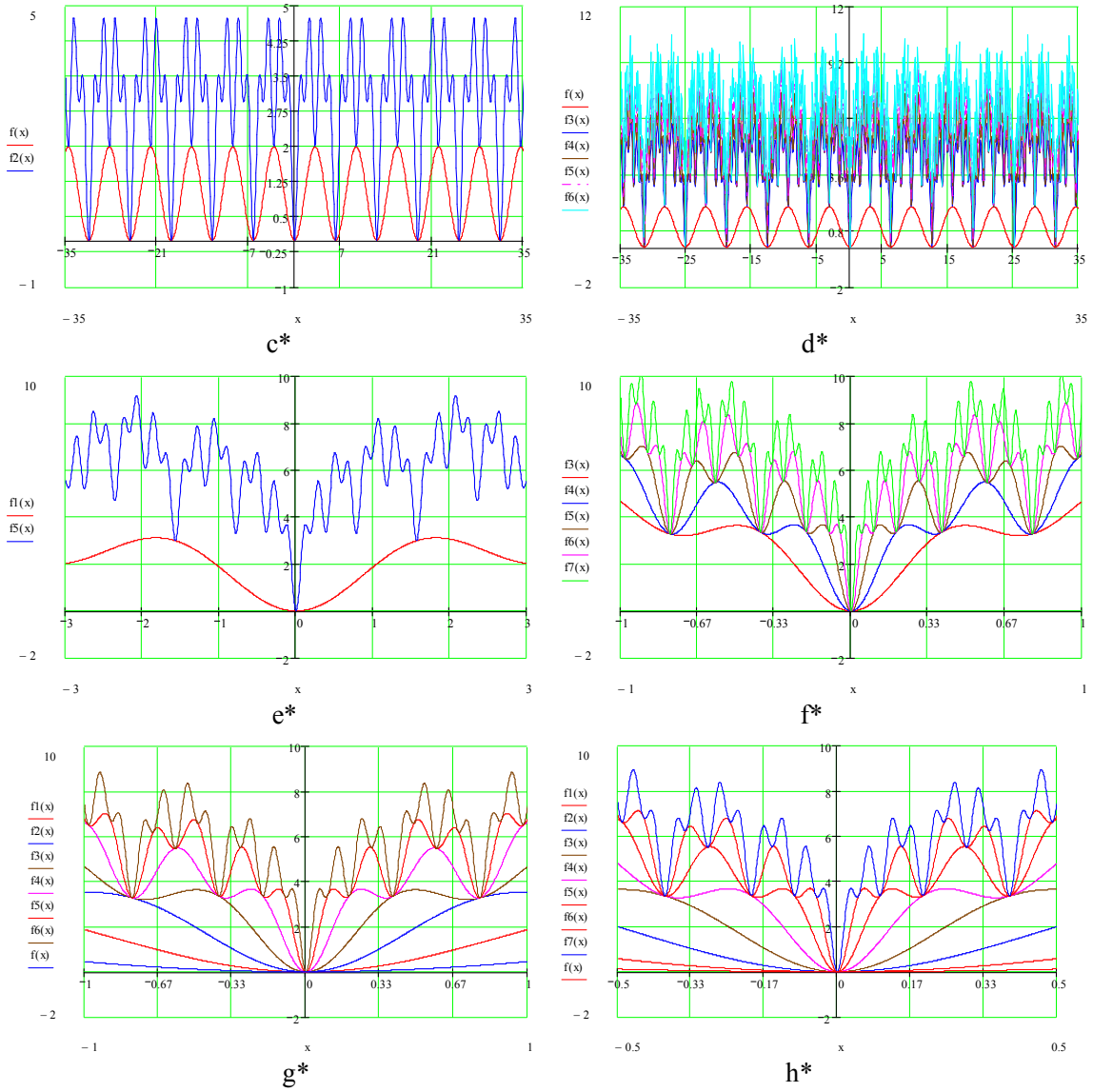


Figure 5. Multistep-coupled heavy rotors nonlinear dynamics vizualizations (Multipliers). Potential energy curves families.

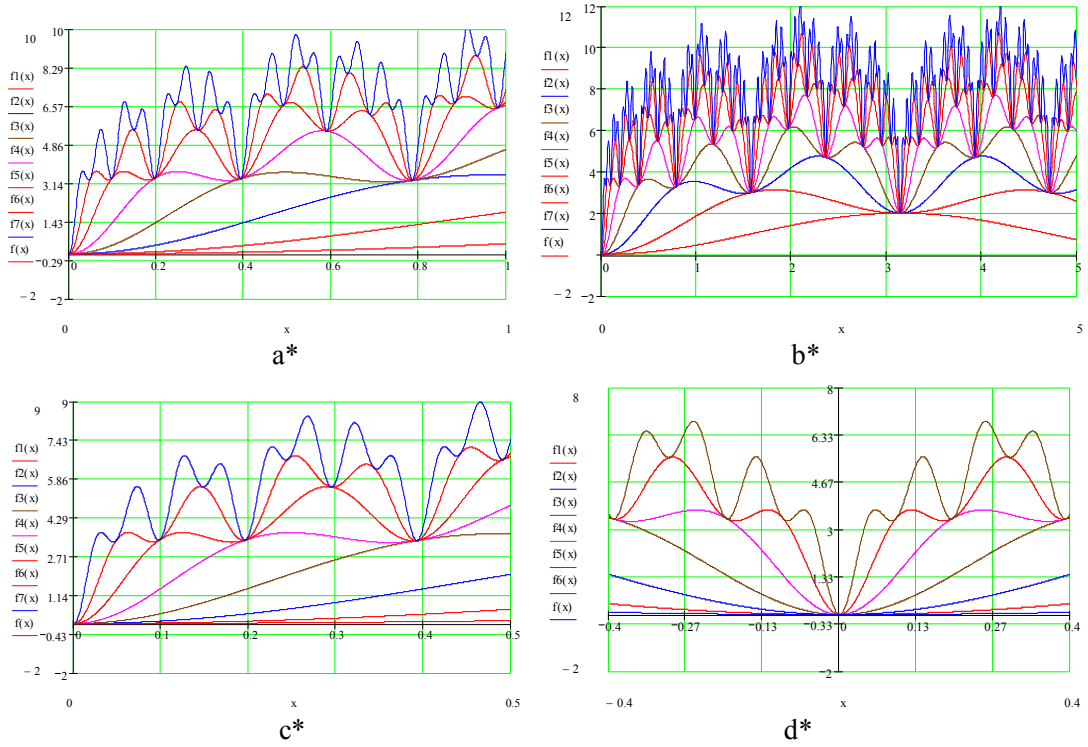


Figure 6. Multistep-coupled heavy rotors nonlinear dynamics vizualizations (Multipliers). Potential energy curves families.

VIII. Angular velocity and angular acceleration of the vector rotators.

Angular velocity of rotator vector $\vec{R}$ rotation about axis of the rotor shaft and angular acceleration are defined by expressions (7).

For the rigid body rotation about fixed axis with constant angular acceleration second order (jerk) is constant $\dddot{\omega} = \dddot{\omega}_0 = const$ and angular acceleration is $\ddot{\omega} = \dddot{\omega}_0 t + \ddot{\omega}_0$, angular velocity $\dot{\omega} = \frac{\dddot{\omega}_0}{2} t^2 + \ddot{\omega}_0 t + \dot{\omega}_0$ and with angle of rotation $\varphi = \frac{\dddot{\omega}_0}{6} t^3 + \frac{\ddot{\omega}_0}{2} t^2 + \dot{\omega}_0 t$. The expression for relative angular velocity and angular acceleration of rotator rotation about axis of rotor shaft for the case with constant angular acceleration second order (jerk) are in the form (see Figure 7.):

$$\dot{\vartheta} = -\frac{2\dot{\omega}_0^2 \left(\dot{\omega}_0 t + \omega_0\right)}{\left(\dot{\omega}_0 t + \omega_0\right)^4 + \dot{\omega}_0^2}, \qquad \ddot{\vartheta} = \frac{2\dot{\omega}_0^3 \left[3\left(\dot{\omega}_0 t + \omega_0\right)^4 - 2\dot{\omega}_0^2\right]}{\left[\left(\dot{\omega}_0 t + \omega_0\right)^4 + \dot{\omega}_0^2\right]^2} \tag{28}$$

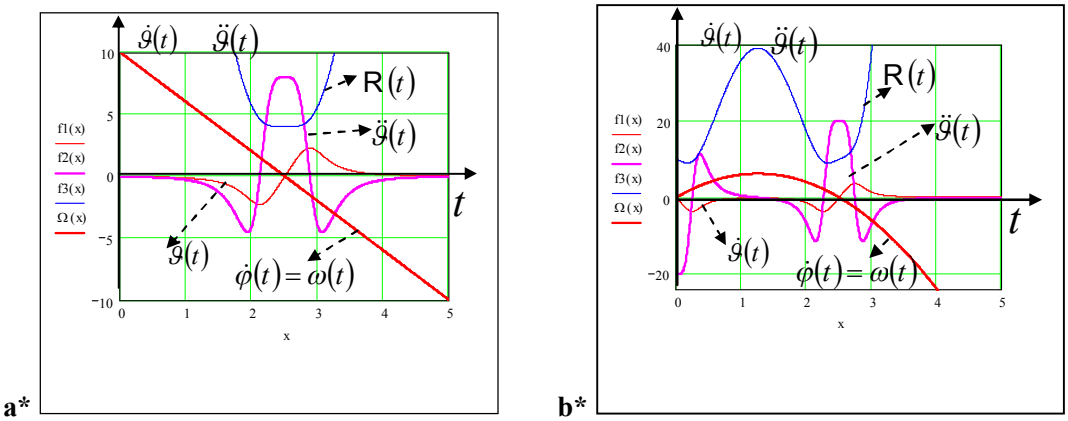


Figure 7. The graphical presentation for relative angular velocity and angular acceleration of rotator rotation about axis of rotor shaft for the case with constant angular acceleration (**a***) and for constant angular acceleration second order (jerk) (**b***) with corresponding graphs of the intensity of vector rotators.

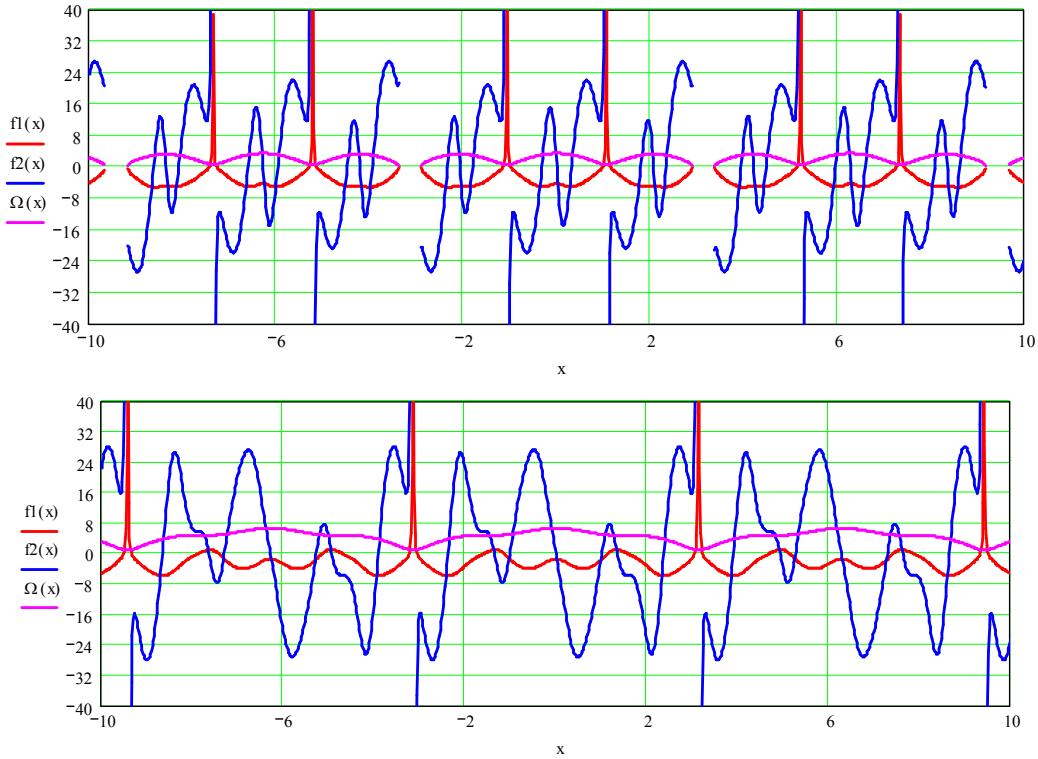


Figure 8. The graphical presentation for relative angular velocity and angular acceleration of rotator rotation about axis of rotor shaft for the case of the double step coupled rotors in the gravitational field for different initial conditions.

IX. Coning Remarks

Mass moment vectors and their deviational components are used to determine kinetic pressures as well as kinetic impacts on axis shaft bearings and compose the systems of the nonlinear differential equations

of coupled rotor dynamics in the earth's gravity field as well as under the action of the impacts of the impulse forces.

It is clearly visible that these two components, are with opposite directions and build **a deviational couple of the kinetic impacts** with intensity $\left|\vec{D}_{A(j)}^{(\bar{n})}\right|\left|\int_{t_0}^{t}\vec{R}_j dt\right|$ to the every pair of the shaft bearings of every of the coupled rotor shafts.

The energy integral is determined, as well as the phase trajectories equations n analytical form for the case of the multi step redactor.

The phenomenon of the layering of homoclinic orbits with an increased number of coupled rotors is determined, so that in two coupled rotors in the earth's gravity field phase portrait a closed homoclinical orbit in the form of number eight and a trigger of coupled singularities composed of three singular points, one unstable saddle type homoclinic points and two stable centre type points appears.

This paper shows, on an example of coupled rotors, the correctness of the trigger of coupled singularities theorem and the existence and non-existence of homoclinic orbits in the form of number eight theorem formulated by the author in the previous published paper. A theorem of layering of homoclinic orbits and of properties of those orbits is also formulated.

The existence of the trigger of coupled singularities and closed homoclinic orbits and that their layering with respect to the change of kinetic parameters of the system is the cause for the appearance of chaotic and stochastic like system behaviour under the influence of purely periodical system excitation, as well as the appearance of sensitivity of system dynamics on small changes of starting initial conditions in the area of homoclinic points and orbits, was shown.

For examine natural clocks of reductor (power transmission), as well as a source of nonlinear vibrations and noise in its dynamics, it is necessary to investigate properties of nonlinear dynamics, and phase portraits, as well as structures of homoclinic orbits, layering and sensitivity of this layering of homoclinic orbits and bifurcation of homoclinic points. In the paper the natural clocks of nonlinear dynamics of coupled rotors are studied.

The Figures represent the graphical presentation of the families of potential energy curves for different steps of multistep coupled heavy rotors dynamics, as reductor or multiplier, of the rotate motions. In the other series of Figures the graphical presentation of the nonlinear dynamics of multistep coupled heavy rotors - reductor or multiplier of the rotate motions by phase plane method is pointed out. In cases shown in the Figures, characteristic homoclinic orbits for one, four or three or sixth step coupled heavy rotors - multiplier for different kinetic parameters of the system are shown.

If we observe a configuration of disbalance of the heavy rotors from the second, third and four rotor's shaft, which are reduced on the first shaft, it can be seen that, in that case, due to the difference in rotation angular velocity of the second, third and four shafts in relation to the first shaft, rotate with different angular velocities. It means also, that the configuration of disbalance of equivalent rotor masses exchange while system is in rotation, which changes the deviation properties of the system, and that brings into system the properties of nonlinearity called first kind, also nonlinearity of second kind, and higher with the periodical character less multiple or more multiple periodical than first nonlinearity depend on the reduction or multiplication of the rotation number of other shafts in relation to the rotation number of the first shaft. By using the previously presented data we can explain the phenomena of bifurcations of the equilibrium positions and phenomena of the trigger of coupled singularities (see Rfs [1, 19]) likee the existence or nonexistence of homoclinic orbits in the form of number eight or of multiplication of number eight in phase portraits. Here we also have coupled

rotations with a change of deviation properties, so we can conclude that in nonlinear dynamics of reductor, coupled rotations bring, under certain constructive parameters of reductor, bifurcation properties of nonlinear dynamics of the second or a higher kind which are the sources of the appearance of chaoticlike and stochasticlike behavior of system dynamics under the excitation with pure periodical forces. At such time, a regime of double period of forced oscillation can appear.

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