OCCURRENCE OF SUBHARMONIC RESONANCE WITH ALLOWANCE FOR DISSIPATION AT POLYHARMONIC EXCITATION

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Abstract

Is established that the additional high-frequency or low-frequency excitation make a essential influence on a level of dissipation, which determining the conditions of occurrence of subharmonic resonance's. As applied to analytical investigation of this problem the efficiency of transformation the initial differential equation to the modified type is established. As this takes place the additional excitation is replaced with energetically equivalent correction of dissipative forces. Fore some characteristics of hysteresis loops the analytical solutions are received that allow to predict the occurrence of subharmonic resonances. The results of computer simulation are in good agreement with the results of analytical investigation.

1.Introduction The essential influence of dissipative forces on the vibrations is observed basically on frequencies close to frequencies of free vibrations. In essence, all resonant phenomena can be considered as free vibrations supported by compelling forces that compensate the negative work of resistance. Such view of the physical nature of a resonance enables in many cases to receive effective analytical dependences allowing to estimate a peak level of researched modes and the conditions of their existence.

The special class of the problems connected to realization of the given approach is devoted to research of the action of additional excitation on the change of effective dissipative forces. For viscous and dry friction this problem was considered by M.Z. Kolovsky [1], and for positional resistance in works [2–7].

In real mechanical systems the dissipative forces practically are always nonlinear; therefore the engineering determination these forces is based on the limited initial information, which is determinate for typical objects with monoharmonic excitation in resonant or free damped modes. As is shown in [2,3], for the cases of biharmonic and polyharmonic excitation the unknown logarithmic decrement would be expressible as $\vartheta=\vartheta_0\Phi(z)$, where ϑ_0 is the volume of ϑ at monoharmonic vibration, $\Phi(z)$ is the factor of decrease with taking into account the additional vibration, z is the ratio between maximal velocities of basic and additional vibrations.

2.Analytical investigation. Let's consider the differential equation, describing the nonlinear forced vibrations in system with one degree of freedom and with the account of additional excitation

$$\ddot{q}+\left|R(q)\right| sign\dot{q}+P(q)=w_1\sin(\Omega_1 t+\varphi)+a\Omega_2^2\sin\Omega_2 t, \qquad (1)$$

where q is the generalized coordinate; $R(q) = -|R(q)| sign\dot{q}$ is the positional dissipative force; $-P(q)$ is the elastic force; the terms of the right-hand side of Eq.(1) correspond to the disturbing forces with frequencies Ω_1 (basic frequency) and $\Omega_2 \ll \Omega_1$ (additional frequency). (All forces are related to unit of mass).

For study the subharmonic vibrations with account of additional excitation not narrowing a generality in the approach to this problem we shall accept $|R(q)| = \zeta |P(q)|$, that corresponds to the triangular loop of hysteresis meeting in many engineering appendices. Besides is concretized the function $P(q)$ as $P(q) = k_0^2(1+\alpha q^2)q$, where k_0 is the frequency of free vibration with $\alpha = 0$. Within the framework of the given paper the accepted elastic characteristic plays a role only of mathematical model, on which example the developed here approach to the decision on the problem are illustrated.

After transition of dimensionless time $\tau = k_0 t$ we shall present the Eq.(1) as

$$q'' + \zeta(1+\alpha q^2)|q| sign q' + (1+\alpha q^2)q = f_1 \sin(\omega_1 \tau + \varphi) + a\omega_2^2 \sin \omega_2 \tau, \qquad (2)$$

where $f_1 = w_1 / k_0^2$, $\omega_1 = \Omega_1 / k_0$, $\omega_2 = \Omega_2 / k_0$, $()' = d / d\tau$.

We shall study the conditions of excitation the subharmonic resonance of the order 1/3. At first, let’s consider the case without additional excitation $(a = 0)$. On the basic of Eq.(2) we have [8]

$$q \approx A \sin \omega_0 \tau + A_3 \sin 3\omega_0 \tau, \qquad (3)$$

where $\omega_0 = 1 + 0,75\alpha A^2$, $\omega_1 = 3\omega_0$, $A_3 = \alpha A^3 / (32 + 21\alpha A^2)$.

Thus the absorbed energy of unit mass in a time of period $2\pi / \omega_0$ can be described as

$$\Delta E_- = 2\zeta^0 (A^2 + 0,5\alpha A^4) k_0^2. \qquad (4)$$

Here ζ^0 corresponds to the case $a = 0$.

The accumulated energy ΔE_+ is equal to the work of disturbing force

$$\Delta E_+ = 3\pi f_1 A^3 \alpha k_0^2 \sin \varphi / (32 + 21A^2). \qquad (5)$$

The condition of existence of subharmonic resonance is defined as $\Delta E_- < \Delta E_+$ $(\varphi = \pi / 2)$. Thus

$$\zeta^0 < \zeta_*^0 = \frac{3\pi\alpha f_1 A}{2(1+0,5\alpha A^2)(32+21\alpha A^2)}. \quad (6)$$

It must be emphasized that as against resonant modes, which amplitude is determined by a level of dissipation, in this case the dissipative forces establish only some barrier of energy, with which overcoming occurrence of subharmonic resonance is possible.

Offered in works [6,7] approaches to the account the dissipation with poly-harmonical excitation are based that on the researched made the effective dissipative force will be formed only with the closed loop of a hysteresis. According to above, the transition from the Eq.(2) to the modified equation is feasible, in which there is no additional excitation with frequency ω_2 but the correction of dissipative component is entered:

$$q'' + \zeta^0(1+\alpha q^2)|q|\eta(|q'| - v|\cos\omega_2\tau|)\,sign q' + (1+\alpha q^2)q = f_1 \sin(\omega_1\tau + \varphi) \quad (7)$$

where η is the unit function; v is the amplitude of dimensionless speed for a harmonic ω_2.

On the ground Eq.(7) with a method of harmonically linearization

$$\zeta^0\eta(|q'| - v|\cos\omega_2\tau|) \to \zeta. \quad (8)$$

Here

$$\zeta = \zeta^0\Phi(z); \quad (9)$$

$$\Phi(z) = \begin{cases} [z\sqrt{1-z^2} - (1-z^2)\arcsin z]/(\pi z^2) & (z \le 1), \\ 1 - 1/(2z^2) & (z > 1). \end{cases} \quad (10)$$

On the Fig.1 the curves $\zeta_*(\omega_1, z)$ with $f_1 = 5$, $\alpha = 0,5$ are given (solid lines). The value ζ_* corresponds to the top border of excitation of subharmonic resonance. The dotted curve corresponds in the other scale to the function $A(\omega_1)$.

The analysis of plots testifies to essential decrease of a level of dissipation with $z < 1$ that results in growth ζ_* and considerably expands the range of frequencies of possible excitation of subharmonic resonance. It is interesting that the frequency ω_1 adequate to the maximal ζ_* practically does not depend on z.

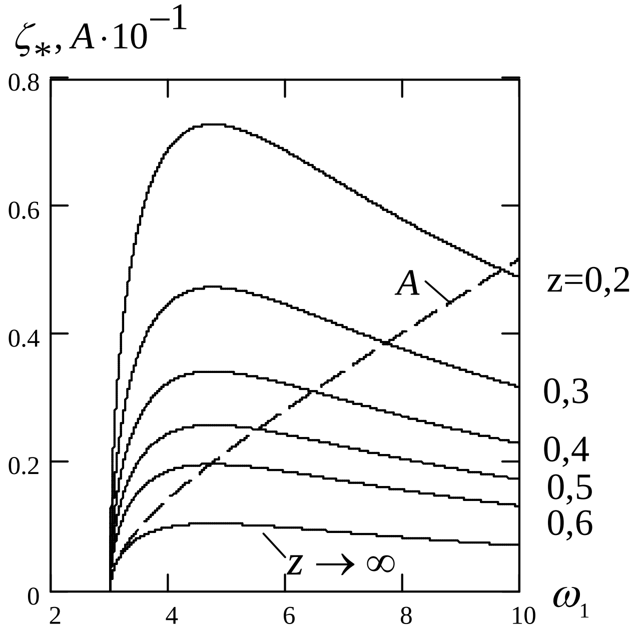

Fig.1

Similar results were received for the elliptic loop of hysteresis. In this case instead of Eq.(7) we have

$$q'' + \frac{\vartheta_0 \Phi(z)\omega_1}{3\pi} q' + k_0^2(1+\alpha q^2)q = f_1 \sin(\omega_1 \tau + \varphi), \tag{11}$$

The function $\Phi(z)$ for the elliptic loop of hysteresis see [3,5].

On the basis of conditions of energy balance it is received $A_2(\omega_1, z) < A \le A_1(\omega_1, z)$.

The limiting values A_1 and A_2 are defined as

$$A_{1,2} = \frac{B_1 \pm \sqrt{B_1^2 - 4B_0 B_2}}{2B_2}, \tag{12}$$

where $B_0 = 32\omega_1^2 \vartheta_0 \Phi(z),\ B_1 = 27\pi f_1 \alpha,\ B_2 = 21\alpha\omega_1^2 \vartheta_0 \Phi(z)$.

On the Fig.2 the family of curves $A_i(\omega_1, z)$ $(i = 1,2)$, at $\vartheta_0 = 0{,}3$ is given.

The range of values appropriate to conditions of existence of a subharmonic resonance is determined by points of crossing of curves A_i and A_* [8], where

$$A_*(\omega_1) = \frac{2\omega_1}{3k_0}\sqrt{\frac{\omega_1^2 - 9k_0^2}{3\alpha}}. \tag{13}$$

On Fig.2 the skeletal curve $A_0(\omega_1)$ also is given.

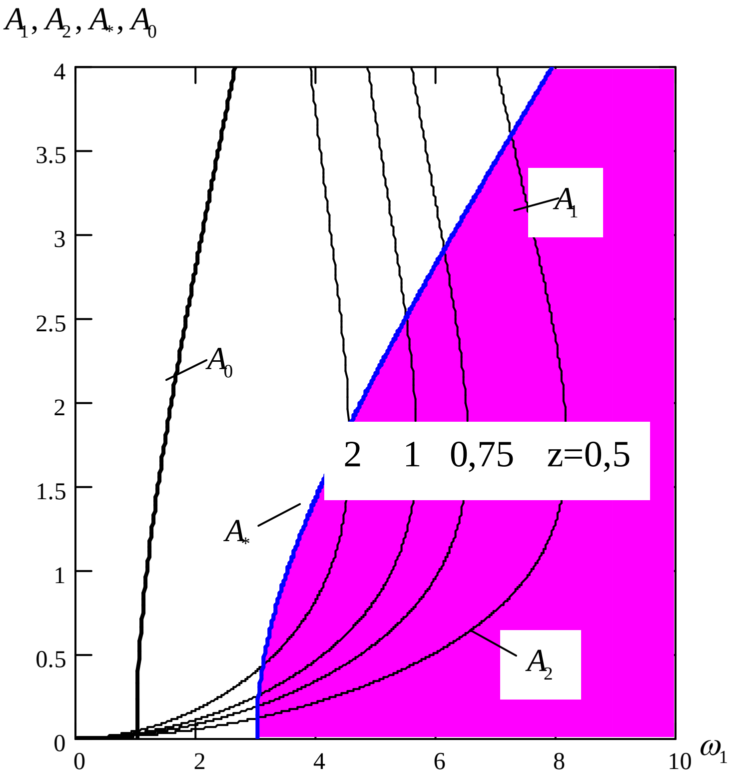

Fig.2

3. Computer simulation. For confirmation of efficiency of the used analytical decision some results received by computer simulation of number modes are given below ($f_1 = 5,\ \alpha = 0{,}5,\ \omega_1 = 6$).

Case 1. $a = 0$ (additional excitation is absent).

Based on the initial differential equation (2) the subharmonic resonance is generated with $\zeta^0 < \zeta_*^0 = 0{,}069$. That is slightly below the result received on the formula (6) because the excitation of subharmonic resonance, and consequently parameter ζ_*^0, usually depends on the initial conditions on which the researched regime can be non-realized. The amplitude of subharmonic resonance is $A = 2{,}82$ that with large accuracy corresponds to a point on the skeletal curve $\omega_0^2 = 1 + 0{,}75\alpha A^2$.

Case 2. $a = 0{,}628;\ \omega_2 = 10$.

On the Fig.3,a the functions $q(\tau)$ and $q'(q)$, resulting from the initial differential equation (2) are plotted with $\zeta_* = 0{,}22$ (This value corresponds to the top border of excitation). The phase trajectory $q'(q)$ testifies to repeated change of speed's sign; that results in decrease the effective dissipation and increase the critical value ζ_*.

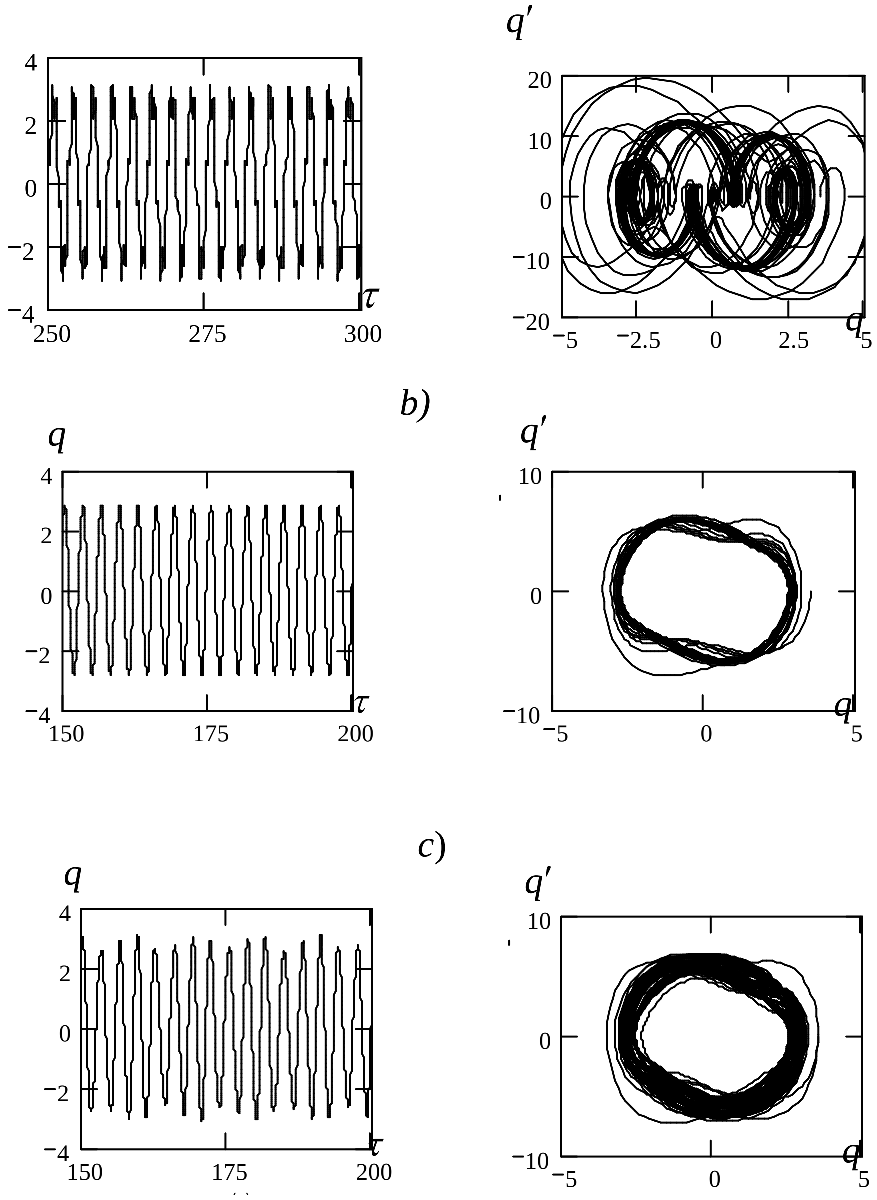

Fig.3

Case 3. $a = 0$; $\omega_2 = 10$; $v = 8,75$.

In this case the solution is obtained on the basis of the modified Eq.(7) (Fig.3,b). As well as in case 2 $\zeta_* = 0,22$. The comparison of cases 2 and 3 shown that the proposed method practically excludes the small influence of a high-frequency component on a peak level, leaving an almost constant the condition of existence of subharmonic resonance.

In this case the phase trajectory $q'(q)$ is «cleared» of high-frequency component with practically invariant amplitude A.

The plots (see Fig.3,b) practically also correspond to the case 1 with absence of additional excitation $(a=0)$, however in this case $\zeta_*^0 = 0,069$ □ ζ_*.

Case 4. $a=0$; $\omega_2 = 0,3$; $v=8,75$.

This case (Fig.3,c) differs from the above in that the addition excitation is low-frequency. Thus $\zeta_* = 0,22$; $A \approx 2,8$, however occur insignificant amplitude-modulated vibrations.

In this case, as differentiated from methods used at high-frequency additional excitation [9], the sufficient remoteness from researched resonant mode is required only. It allows to analyze as well cases when $\omega_2 < 1$ $(\Omega_2 < k_0)$.

The analysis a great body of modes supports the conclusion that the analytical forecasts are in good agreement with the results of computer simulation.

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