MOTION OF AXLE – BOX BETWEEN DELIMITERS OF VIBRATING PENDULUM

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1. Introduction

Investigation include mechanical model of axle – box 2 that move in relative motion along of pendulum *1* road (Fig. 1.). Axe – box and pendulum is connected together with spring and have bumper. System may be excited by kinematics motion of pendulum axe in two directions *Ox* and *Oy*. Additional forces moment around pendulum end point may be added. System investigation may includes two tasks: - possibility of pendulum vibrating motion damping; - use of system for technological needs. Pendulum vibrating motion is investigated for case when kinematics motion of axe is harmonically. In this case many kinds of nonlinear motions of system exist that is known in nonlinear dynamics. By choice of systems parameters (gap, stiffness of spring, mass of axle – box, initial position of axe – box location) efficiency of pendulum vibrating motion damping may be obtained. For technological needs system was evaluated by shock interaction between axle – box and pendulum. It is shown that additional forces moment around pendulum axe gives positive effect.

2. Description of mechanical system

The investigated system in vertical plane has two degree of freedom: rotation motion φ of pendulum *1* (together with axe – box) around it vibrating end point *O* and additional relative translation motion r of axe – box *2* mass centre C_2.

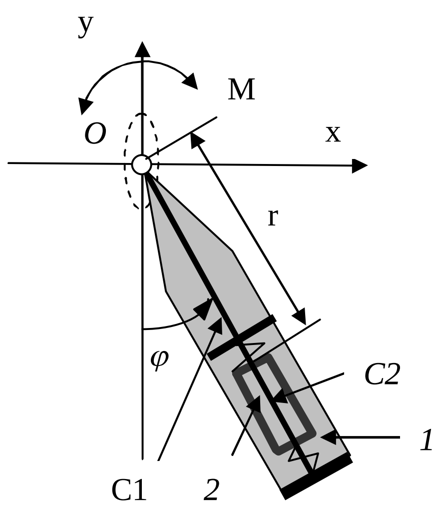


Fig. 1. Scheme of system.

The equations of motion are next (1), (2) [1, 2]:

$$Jo(t)\cdot\ddot{\varphi} = -K_{12}(t)\cdot\left[(g+\ddot{\eta})\cdot\sin\varphi + \ddot{\varsigma}\cdot\cos\varphi\right] - 2\cdot m_2\cdot\dot{\varphi}\cdot\dot{r}\cdot r - b\cdot\dot{\varphi} + M_O^{(e)}; \qquad (1)$$

$$m_2\cdot\ddot{r} = -f_{12}(r,\dot{r}) + m_2\cdot\dot{\varphi}^2\cdot r + m_2\cdot(g+\ddot{\eta})\cdot\cos\varphi - m_2\cdot\ddot{\varsigma}\cdot\sin\varphi. \qquad (2)$$

Here

$$Jo(t) = Jc_1 + m_1\cdot L^2 + Jc_2 + m_2\cdot r^2; \qquad (3)$$

$$K_{12}(t) = Jc_1 + m_1\cdot L^2 + Jc_2 + m_2\cdot r^2. \qquad (4)$$

Were $\dot{\varphi},\ddot{\varphi}$ - angular velocity and angular acceleration both objects; $\dot{r},\ddot{r}$ - derivation in time of distance r (or relative velocity and relative acceleration of axe – box mass centre); m_1, m_2 - masses; Jc_1, Jc_2 - axial moments around centre of masses; L - constant distance from masse centre C_1 to pendulum vibrating end; g - acceleration of free fall; $\ddot{\varsigma},\ddot{\eta}$ - horizontal and vertical acceleration components of vibrating pendulum axe; b - constant of linear damping; $M_O^{(e)}$ - external additional forces moment around pendulum axe; $[-f_{12}(r,\dot{r}]$ - internal interaction forces from technological process. Example of internal interaction inside free motion modeling block is shown in Fig. 2, Fig. 3.

3. Motion simulation

First series of modeling was made as free motion to check main model without excitation. Results of investigations are shown in Fig. 2. - Fig. 5. Comments are given under any picture.

$$\begin{bmatrix} r_{n+1}\\ v_{n+1}\\ \varphi_{n+1}\\ \omega_{n+1}\end{bmatrix} := \begin{bmatrix} r_n + s\cdot v_n \\ v_n + \frac{s}{m2}\cdot\left[\left[m2\cdot r_n\cdot(\omega_n)^2 + m2\cdot(g)\cdot\cos(\varphi_n)\right] - c\cdot(r_n-\Delta) - c2\cdot(r_n-\Delta1)\cdot\left(0.5-0.5\cdot\frac{\Delta1-r_n}{|\Delta1-r_n|}\right)\cdot\left(0.5+0.5\cdot\frac{v_n}{|v_n|}\right)\right] \\ \varphi_n + s\cdot\omega_n \\ \omega_n + \frac{s\cdot\left[-(m1\cdot L+m2\cdot r_n)\cdot\left[(g)\cdot\sin(\varphi_n)\right] - 2\cdot m2\cdot\omega_n\cdot v_n\cdot r_n - b\cdot\omega_n\right]}{Jc1+m1\cdot L^2+Jc2+m2\cdot(r_n)^2}\end{bmatrix}$$

Fig. 2. Example of internal interaction inside modeling block. Interaction depends of displacement and velocity as linear loop.

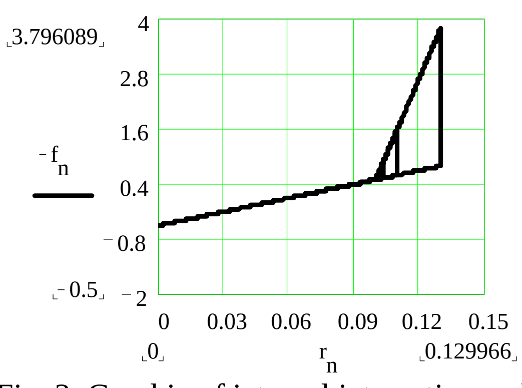


Fig. 3. Graphic of internal interaction and displacement. Interaction depends of displacement and velocity as linear loop.

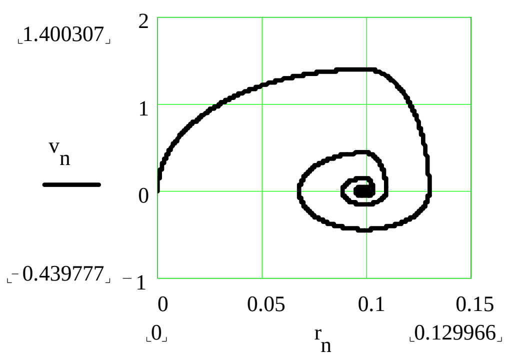


Fig. 4. Axe – box damping motion in phase plane. After three impacts internal interaction is stopped.

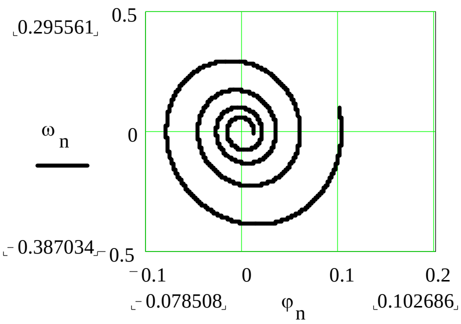


Fig. 5. Motion of pendulum in phase plane by linear damping.

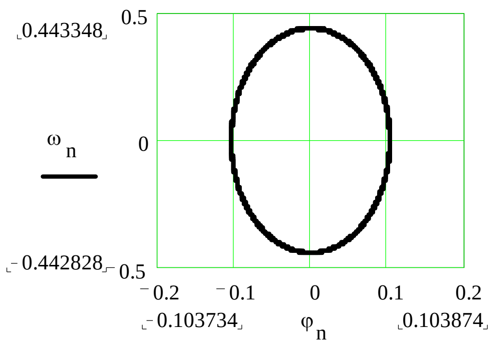


Fig. 6. Motion of pendulum in phase plane with out external damping (b = 0). Carioles force practically does not damped motion.

Second series of modeling was made for only vertical harmonica excitation. Results of modeling are shown in Fig.7. – Fig. 11.

$$\begin{bmatrix} r_{n+1}\\ v_{n+1}\\ \varphi_{n+1}\\ \omega_{n+1}\end{bmatrix} := \begin{bmatrix} r_n + s\cdot v_n \\ v_n + \frac{s}{m2}\cdot\left[\left[m2 r_n\cdot(\omega_n)^2 + m2(g+Ay\cdot\sin(k\cdot n\cdot s))\cdot\cos(\varphi_n)\right] - c\cdot(r_n-\Delta) - c2\cdot(r_n-\Delta1)\cdot\left(0.5-0.5\frac{\Delta1 - r_n}{|\Delta1-r_n|}\right)\cdot\left(0.5+0.5\frac{v_n}{|v_n|}\right) - b1\cdot v_n\right] \\ \varphi_n + s\cdot\omega_n \\ \omega_n + \frac{s\cdot\left[-(m1 L + m2 r_n)\cdot\left[\left[(g+Ay\cdot\sin(k\cdot n\cdot s))\cdot\sin(\varphi_n)\right]\right] - 2\cdot m2\cdot\omega_n\cdot v_n\cdot r_n - b\cdot\omega_n\right]}{Jc1 + m1\cdot L^2 + Jc2 + m2\cdot(r_n)^2}\end{bmatrix}$$

Fig. 7. Equation for modeling with vertical harmonica excitation.

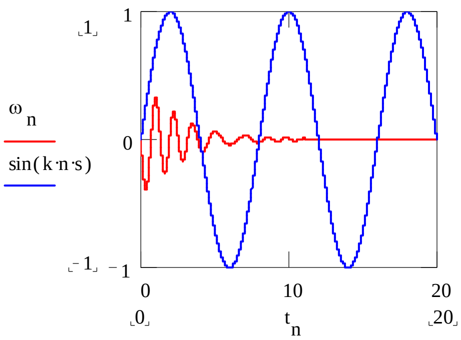


Fig. 8. Angular velocity of pendulum compare with harmonica in time domain.

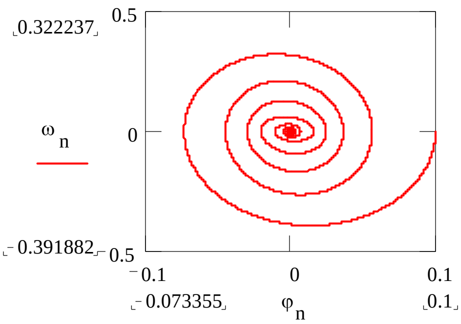
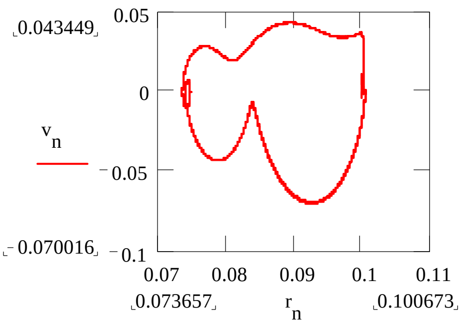


Fig. 9. Motion in phase plane for pendulum (see Fig. 8.).



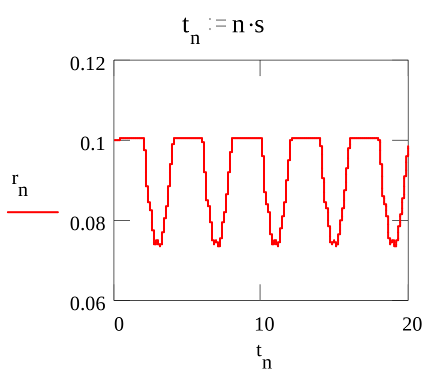


Fig. 10. Motion of axe – box in phase plane (see Fig. 11.).

Fig. 11. Displacement of axe – box in time domain. Stick moment with bumper is seeing (see Fig. 10.).

Third series of modeling was made for common vertical and horizontal harmonica excitation. Results of modeling are shown in Fig.12. – Fig. 15.

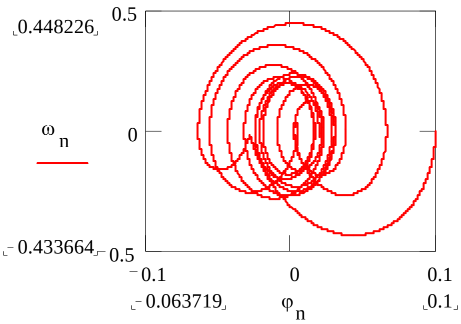


Fig. 12. Motion of pendulum in phase plane when two external components acted.

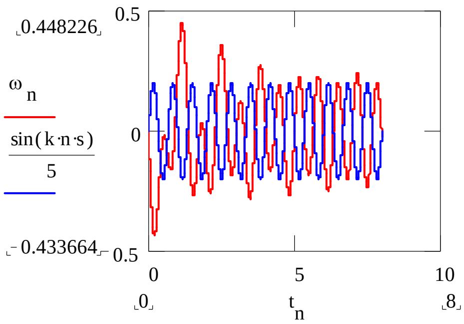


Fig. 13. Angular velocity in time domain and harmonica of excitation.

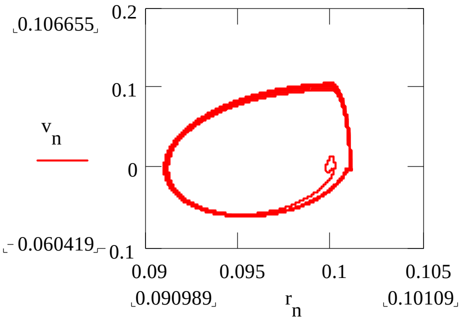


Fig. 14. Relative motion of axe – box in phase plane. Periodic cycle is very stable. It means that this regime may be used for technological processes.

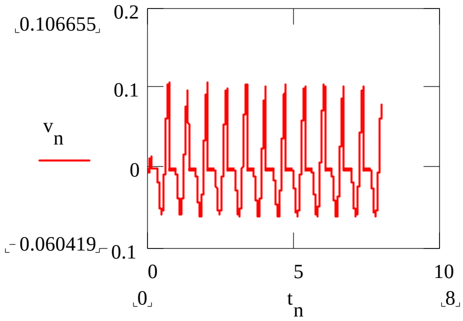


Fig. 15. Relative velocity of axe – box in time domain when two external components are added (see Fig. 12. – Fig. 14.).

Fourth series of modeling was made for common vertical and adaptive moment $(M_O^{(e)} = M \cdot \dfrac{\dot{\varphi}}{|\dot{\varphi}|})$ excitation. Results of modeling are shown in Fig.16. – Fig. 19.

Final series of modeling was made for only horizontal excitation (Fig. 20. – Fig. 21.) and only adaptive moment excitation (Fig. 22., Fig. 23.).

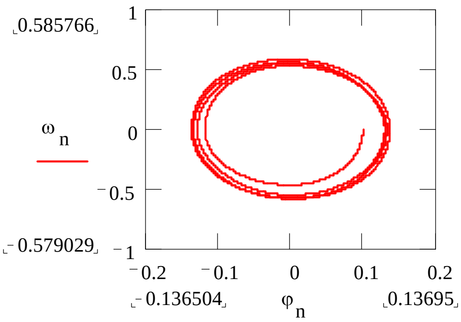


Fig. 16. Motion of pendulum in phase plane when vertical excitation exists and additional

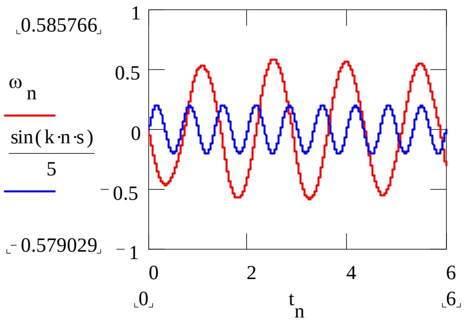


Fig. 17. Motion of pendulum in time domain when vertical excitation exists and additional

adaptive moment ($M_O^{(e)} = M \cdot \frac{\dot{\varphi}}{|\dot{\varphi}|}$) is added.

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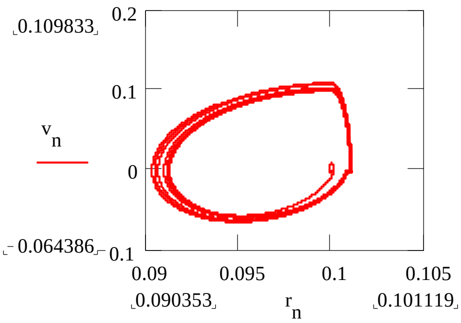

Fig. 18. Relative motion of axe – box in phase plane when vertical excitation exists and additional adaptive moment is added.

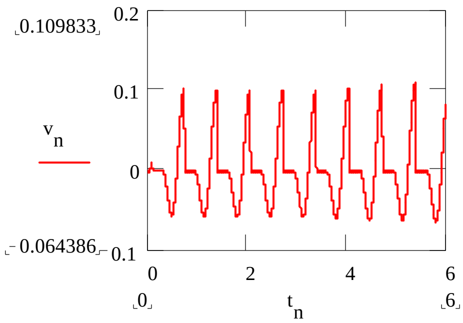

Fig. 19. Relative velocity of axe – box in time domain when vertical excitation exists and additional adaptive moment is added. (see Fig. 16. – Fig. 18.).

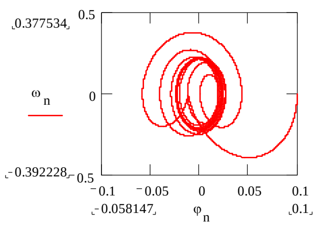

Fig. 20. Motion of pendulum in phase plane when only horizontal excitation exists. System is useless for technological processes.

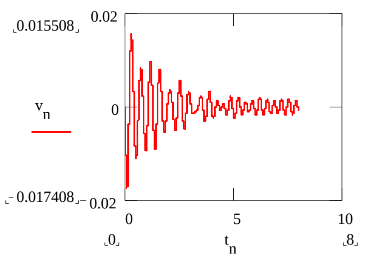

Fig. 21. Motion of axe - box in time domain when only horizontal excitation exists. System is useless for technological processes.

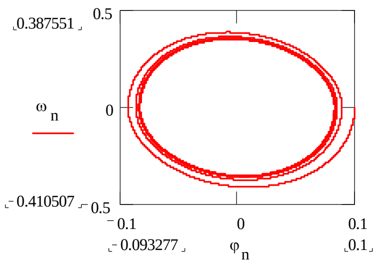

Fig. 22. Motion of pendulum in phase plane when only moment of excitation exists. System is useless for technological processes.

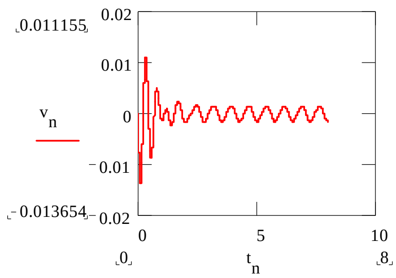

Fig. 23. Motion of axe - box in time domain when only moment of excitation exists. System is useless for technological processes.

Conclusion

Pendulum vibrating motion is investigated for the case when kinematics motion of axe is harmonically and additional force moment may be added. For technological needs vertical exciting together with horizontal exciting or force moment must be used. Case with only one excitation is small efficiency. Tasks of damping possibility needs more investigations.

References

1. O. Kepe, J. Viba. Teoretiska mehanika. Zvaigzne. 1982.Riga. Latvija. - 577. lpp.
2. Я. А. Виба. Оптимизация и синтез виброударных систем. Зинатне. 1988. Рига. - 253. стр.

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