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INVESTIGATION OF STICK-SLIP FRICTIONAL SELF-EXCITED VIBRATIONS WITH ACTION OF HIGH-FREQUENCY DISTURBANCES

Iosif I. Vulfson
St. Petersburg State University of Technology and Design, St-Petersburg, Russia
jvulf@yandex.ru

Abstract. *We consider the problem when the sliding speed of the slider has a high frequency component. It was determined the influence of this component on the critical sliding velocity, below which the frictional stick-slip self-excited vibrations are generated. The criterion for determining the optimum of dynamic characteristics of the system is suggested.*

Key words: frictional self-excited vibrations**,** high frequency excitation**,** critical sliding velocity.

ИССЛЕДОВАНИЕ РАЗРЫВНЫХ ФРИКЦИОННЫХ АВТОКОЛЕБАНИЙ ПРИ ВЫСОКОЧАСТОТНЫХ ВОЗМУЩЕНИЯХ

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Федеральное государственное бюджетное образовательное учреждение высшего профессионального образования Санкт-Петербургский государственный университет технологии и дизайна, Санкт-Петербург, Россия.

Аннотация *Исследуется задача о перемещении ползуна при скорости скольжения, которая содержит высокочастотную составляющую. Установлено, что эта составляющая может существенно влиять на критическую скорость, ниже которой возбуждаются разрывные фрикционные автоколебания. Предложен критерий для выбора оптимальных динамических характеристик системы.*

Ключевые слова: *фрикционные автоколебания; высокочастотное возбуждение; критическая скорость скольжения.*

When operating machinery the frictional oscillations occur most often under slow movement of the sliding boxes on guides or under shaft's rotation at small angular velocities. In particular the movement with periodic stops can occur in machine tools when moving heavy units instead of the required uniform motion. This exclude the possibility of precise tool feed. A similar phenomenon is observed in the drawing rollers of the spinning machines, in which the frictional self-oscillation causes yarn unevenness and increases its breakage. The problem of frictional self-oscillations is the subject of many studies, a review of which is not given here [1]. We confine ourselves to the brief information on the conditions of excitation of frictional self-excited oscillations and taking into account the influence of high-frequency disturbances on these conditions [2- 9].

Let us turn to the model shown in Fig. 1.

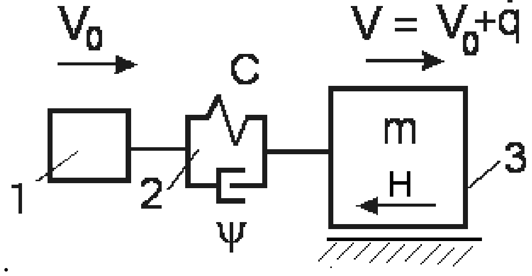

Fig. 1. Dynamic model

Let us imagine that from link 1, moving with constant velocity $v_0 > 0$, motion through elastic dissipative element 2 is transmitted to unit 3, having mass m, to which the force of friction H is applied. As the generalized coordinate, we accept the deformation of the elastic element q. Then, the velocity of the link 3 is equal to $v = v_0 + \dot{q}$. Obviously, under $H = H_0 = \text{const}$, we have $q = -H_0 / c = \text{const}$, and therefore $v = v_0$. However, the frictional force depends on many factors, including sliding velocity and, therefore, in general, $H(v) \neq \text{const}$ at that $\dot{q} \neq 0$. We assume simplified characteristics of the frictional forces, according to which the absence of sliding $(v = 0)$, frictional force is equal to the force of static friction $H = H_0$ and under motion $(v > 0)$ $H = H_1 < H_0$.

Let us write the differential equation for the phase of mass m movement $(0 < t < t_1)$

$$m\ddot{q} + b\dot{q} + cq = -H_1, \qquad (1)$$

where $b = \psi\sqrt{cm/(2\pi)}$ is the factor of equivalent linear resistance; ψ is the dissipation factor. Transform (1) to the form

$$\ddot{q} + k^2 q = -H_1 / m - 2n\dot{q}.$$

Here $k^2 = c/m$; $2n = b/m$.

Let's input "dimensionless time " $\varphi = kt$. Then

$$q'' + q = \Delta(q'), \qquad (2)$$

where $\Delta = -F_1 / c - 2\delta q'$; $\delta = n/k = \vartheta_0 /(2\pi)$; the derivative with respect to φ is denoted with prime mark.

The phase-plane portrait shown in Fig. 2, for several typical cases, corresponds to (2). Let's consider first of all the case, when there is no linear resistance $(\delta = 0)$. At the same time in the area of motion, the phase trajectory is presented as a circle centered at the point $\Delta_* = -H_1 / c$ (Fig.2, curve 1).

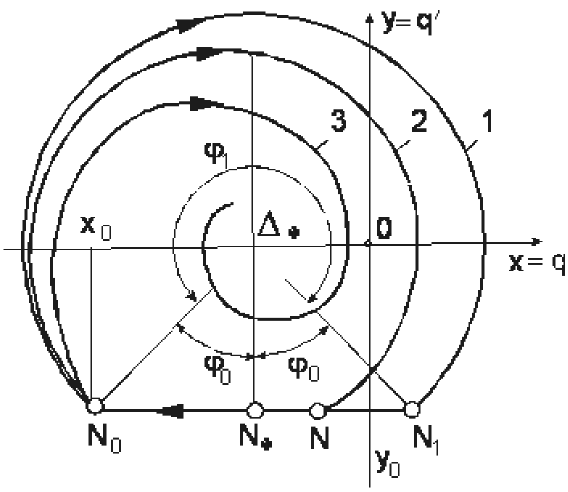

Fig. 2. Phase-plane portrait

Let's establish initial conditions. The movement begins only when the restoring force balances the force of static friction. When $t = 0$ we have $q_0 = -H_0 / c$. The second initial condition is determined on the basis of the obvious equality $v = v_0 + \dot{q}_0 = 0$; hence $\dot{q}_0 = -v_0$. The point N_0

with coordinates $x_0 = q_0$ and $y_0 = q_0' = -v_0 / k$ corresponds to the initial conditions in the phase plane.

Movement of mass will continue as long as the phase trajectory will not come to the point N_1. The value $v = v_0 + \dot{q} = 0$ corresponds to this point, so the frictional force becomes equal to the force of static friction H_0. In the zone of dwell $y = -v_0 / k = \text{const}$, the plot phase trajectory is a straight line N_1N_0. Further, the above-described oscillatory process is repeated. Thus, uniform motion of input link 1 transformed into relaxation oscillations with dwells (stick-slip motion). The area of movement corresponds to the angle $\varphi_1 = 2(\pi - \varphi_0)$ and time interval $t_1 = \varphi_1 / k$ [corresponds to the area of movement]. The angle φ_0 is defined as

$$\varphi_0 = \arctan\left|(x_0 - \Delta_*)/ y_0\right| = \arctan[k\Delta H /(cv_0)]\,. \qquad (3)$$

Here $\Delta H = H_0 - H_1$ is the difference between static and sliding friction.

We will find the period of self-excited oscillations $\tau = t_2$, from the obvious equality of the distances passed with the link 1 and mass m for one cycle

$$\Delta s = v_0\tau = v_0 t_1 + q(t_1) - q_0\,,$$

where $q(t_1) - q_0 = N_0N_1 = 2\Delta H / c$.

Considering, $t_1 = \varphi_1 / k = 2(\pi - \varphi_0)/ k$, we get

$$\tau = 2(\pi - \varphi_0)/ k + 2\Delta H /(cv_0) \qquad (4)$$

The dimensionless value of the period of self-excited oscillations is equal to

$$\varphi_2 = k\tau = 2\pi + 2(\tan\varphi_0 - \varphi_0)\,. \qquad (5)$$

It follows from (3)–(5), that when $\Delta H \to 0$, we have $\varphi_0 \to 0$, $\tau \to 2\pi / k$, $\varphi_2 \to 2\pi$.

The amplitude of oscillations A_0 is determined as

$$A_0 = \sqrt{x_0^2 + y_0^2} = \sqrt{(\Delta H / c)^2 + (v_0 / k)^2}\,. \qquad (6)$$

Next, we take into account the effect of the linear resistance, the influence of which will manifest itself in the fact that the amplitude will decrease by the law $A = A_0 e^{-nt}$. At the same time the phase trajectory (curve 2) is located within a circle 1 of radius A_0, and the dwell begins at the point N. The graph $\dot{q}(t)$ shown in Fig. 3, a corresponds to this case. However it may be, that $v_{\min} = v_0 + \dot{q}_{\min} > 0$ and therefore $\dot{q}_{\min} > -v_0$. Then the area of dwell is absent and oscillations will attenuate by exponential law (Fig. 3, b). At that the phase trajectory takes the form of curve 3 (see Fig. 2).

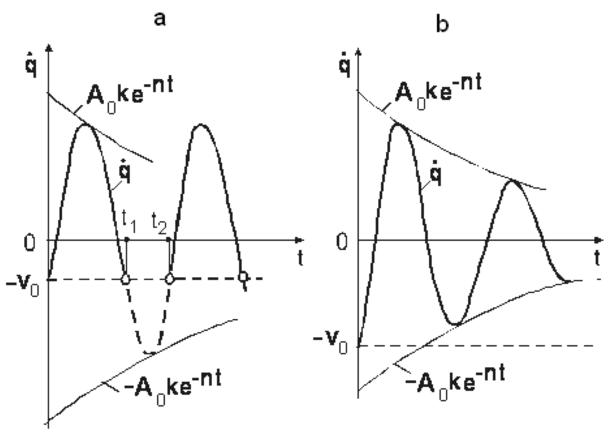

Fig. 3. To the determination of the dwell area

. The critical value of v_0^* , which separates these two cases, corresponds to the point of touching of the phase trajectory with the straight line N_0N_1. The analysis shows that the influence of linear dissipation on the position of the point contact is negligible and it is located in the vicinity of N_*. At that

$$A_0 k \exp(-nt_1^*) = v_0^*, \tag{7}$$

where, $t_1 = t_1^* = (2\pi - \varphi_0^*)/k$. (The parameters values corresponding to this critical case are marked with the asterisk.)

On the basis of (7) and phase-plane portrait analysis, we get

$$\cos\varphi_0^* = v_0^*/(kA_0) = \exp[-\vartheta_0(1-\varphi_0^*/2\pi)].$$

Here ϑ_0 is the logarithmic decrement

It follows from here that

$$\vartheta_0(1-\varphi_0^*/2\pi) = -\ln(\cos\varphi_0^*). \tag{8}$$

On the basis of (8) when $0.2 \le \vartheta_0 \le 0.8$, we get $\varphi_0^* \approx [\pi/6 \div \pi/3]$.
Next, using (6), (7), we finally get

$$v_0^* = \frac{k\,\Delta H}{c\sqrt{\exp(2\vartheta_1)-1}} = \frac{k\,\Delta H}{c}\sqrt{\frac{1-\psi_1}{\psi_1}}, \tag{9}$$

where, $\vartheta_1 = \vartheta_0(1-\varphi_0^*/(2\pi) \approx [5/6 \div 11/12]\vartheta_0$; $\psi_1 = 1-\exp(-2\vartheta_1)$ are the corrected values of logarithmic decrement ϑ_0 and dissipation factor ψ_0.

Taking into account the degree of validity of the initial information and smallness of the corrected modification, we can accept $\vartheta_1 \approx \vartheta_0$; $\psi_1 \approx \psi_0$.

It should be emphasized that it is the dissipative properties of the drive, which determine the final value of the v_0^*, since when $\psi_0 \to 0$, we have $v_0^* \to \infty$. This means that, excluding the linear resistance, we can see that the self-excited oscillations of the considered type can occur at any velocity V_0. Typically, the critical velocity ν_0^* is sufficiently small. Suppose, for example, $k = 100\,\mathrm{s}^{-1}$, $\Delta f = 0.05$; $\vartheta_0 = 0.2$. At that $\varphi_0^* = 0.584$ rad ; $\nu_0^* = 0.741\cdot 10^{-2}$ m/s.

As it was already mentioned, while designing machine tools, devices and other equipment, occurring in case of frictional self-oscillations, transformations of uniform movement into stick-slip movement (i.e. motion with dwells and jumps) is highly undesirable. The point is the value of jump Δs ultimately determines the so-called positioning accuracy, i.e. realized accuracy of the executive body in achieving the specified position.

We consider two critical cases for estimation of Δs. In case of velocity v_0, nearest to the critical value of v_0^*, we have $\varphi_1^* \approx 2\pi - \varphi_0^*$, where $\varphi_0^* = \varphi_0(v_0^*)$. Then

$$\Delta s_* = \nu_0^*(2\pi - \varphi_0^*)/k + q(t_1) - q_0.$$

In this case $\Delta q = q(t_1) - q_0 \approx N_* N_0 = \nu_0^* k^{-1} \tan\varphi_0^* = \Delta H/c$. It follows that

$$\Delta s_* = (\nu_0^*/k)[2\pi - \arctan(k\Delta H)/(c\nu_0^*)] + \Delta H/c.$$

The period $\tau_* = \Delta s_* / \nu_0^*$ corresponds to this jump. When velocity ν_0 transits through critical value of ν_0^* period decreases with a jump to the value $\tau = 2\pi / k$, i.e. by the value of $\Delta\tau_* = (\tan\varphi_0^* - \varphi_0^*)/k$. Under accepted initial data, we have $\varphi_0^* = 0.584$ rad; $\Delta s_* = 0.471$ mm; $\tau_* = 6.36 \cdot 10^{-2}$ s; $\Delta\tau_* = 7.68 \cdot 10^{-4}$ s.

The other critical case is realized when $\nu_0 / k < \Delta H / c$. Such a situation occurs in case of small driving velocity. In this case $\varphi_0 \to \pi/2$. Then, in accordance with (4)

$$\left.\begin{aligned} \tau &= \pi/k + 2\Delta H/(c\nu_0) \approx 2\Delta H/(c\nu_0); \\ \Delta s &= \pi\nu_0/k + 2\Delta F/c \approx 2\Delta H/c. \end{aligned}\right\} \qquad (10)$$

In this critical case, we see the clear manifestation of another curious feature of frictional self-oscillations. As it follows from (10), the period of self-excited oscillation, now, is practically determined by the time interval of mass dwell, when the deformation of the elastic element takes place. Further mass almost instantly "jumps" to the value of Δs.

At that the oscillations turn out to be essentially discontinuous. The malleable drive in this case acts as the energy storage device and appears to be a kind of mediator between the external source and the oscillatory system. As value of ν_0 approach the critical value of ν_0^* the frictional form of self-oscillations becomes less pronounced.

Further we will assume that the speed, of sliding along the guides, has a high-frequency component $v + a\omega\sin\omega t$, where a is the amplitude of oscillations ($\omega \gg k$). This situation in particular occurs under the high frequency vibration of the entire system, which leads to the additional kinematic excitation of the moving mass. Maximum of velocity oscillations, with a frequency k is determined by the dependence $v_k \approx k\sqrt{(\Delta H/c)^2 + (v/k)^2}$.

When $\omega \gg k$ we have $z \approx v_k/(a\omega)$, and at high frequencies ω, even very small amplitudes of the high-frequency excitations a can lead to a significant decrease in the value of parameter z and functions $\Phi(z)$ (Fig.4) [3–5]. In the considered model the difference between frictional forces $\Delta H_z = \Delta H \Phi(z)$ and dissipation factor $\psi_z = \psi_0 \Phi(z)$ is corrected simultaneously; here index z corresponds to taking into account the high-frequency oscillations.

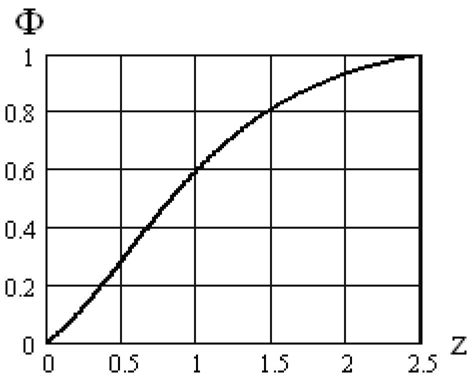

Fig. 4. Graph $\Phi(z)$

On the basis of (9)

$$v_{**} = \frac{k\Delta H}{c}\sqrt{\Phi(1 - \psi_0\Phi)/\psi_0}\,, \qquad (11)$$

where, v_{**} is the critical velocity, taking into account the high-frequency component of oscillations.

Equation (11) shows the opposite trend with decreasing Φ. This is related to the fact that, the efficiency value of the differential frictional forces ΔH_z and the dissipation factor ψ_z, of the drive ψ_z, decrease simultaneously. The first of these factors leads to a decrease in v_*, and the second - to increase. In Fig. 5,a is shown the graphs $f(z,\psi_0) = v_{**}/w$ and the locus of points $\max f(z)$.

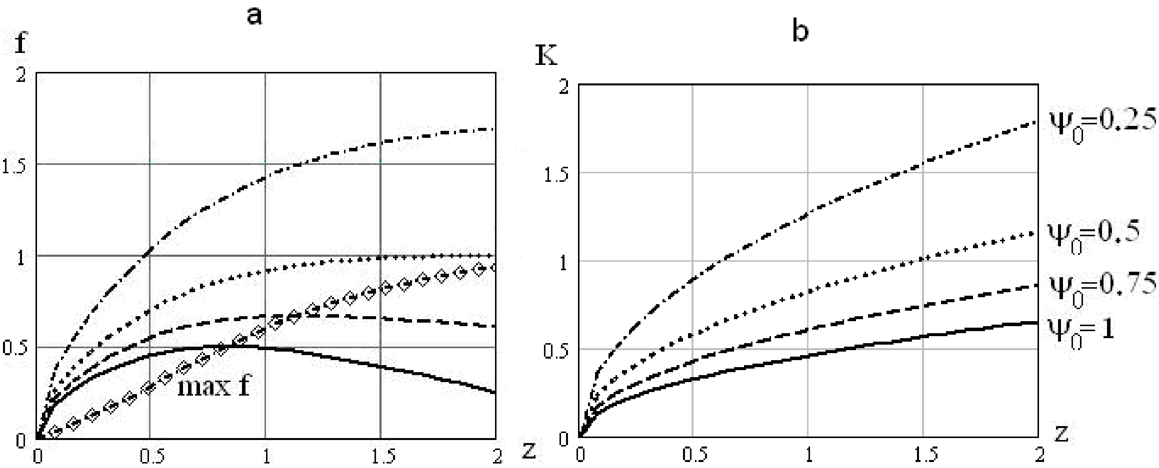

Fig. 5. Graphs $f(z,\psi_0)$, $K(z,\psi_0)$

On the basis of (9) and (11) $v_{**} = v_* K(\Phi)$, where the correction function $K(\Phi)$ has the form

$$K(\Phi) = \sqrt{\Phi(1-\psi_0\Phi)/(1-\psi_0)}. \quad (12)$$

According to (12) $\max K(\Phi)$ corresponds to the condition $\Phi = \Phi_* = 1/(2\psi_0)$. As Φ<1 when ψ_0<0.5 $\Phi \in [0, \Phi_*]$ function $K(\Phi)$ on the whole interval [0,1] is increasing. When ψ_0> 0.5, $\Phi \in [0, \Phi_*]$ the function $K(\Phi)$ increases, and when $\Phi \in [\Phi_*, 1]$ it decreases. . Figure 5,b shows a family of curves $K(z,\psi_0)$.

The analysis shows that to reduce the critical speed of excitation of self-oscillations, more than two times, when $\varphi_0 > 0.25$, it is necessary that $\Phi \le 0.2$, at that $z \le 0,4$. Similar dynamic effects can also occur in case of the high-frequency excitation in the direction perpendicular to the plane of motion [9]. One way to eliminate the identified "jumps" is to use the materials with low difference between coefficients of friction in rest and motion, such as filled fluoroplasts (PTFE, teflon, flourlon) paired with tempered steel. Of course, a more radical way is to start using [motors] drives with ball bearings, in which the sliding friction is completely eliminated. However, in this case usually the reduced stiffness of the drive is decreased and the lower natural frequency increases, which is related to the elimination of the effect of self-braking, which in the given case acts positively, reducing the “length” of kinematic chain of the drive, which is prone to excited oscillations.

In conclusion, we will note that the occurrence of differential friction forces ΔH, according to modern ideas, is treated as feedback broadband random disturbances, arising during sliding of rough deformable bodies. The friction itself, strictly speaking, is formed in an oscillating system directly by the locally occurring dynamic processes. Therefore the use in the engineering calculations the quasi-static friction characteristics is approximate.

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